

## SOLUTION OF A FUNCTIONAL INTEGRAL INCLUSION IN BANACH SPACE

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**Abstract.** In this paper we study the existence of weak solutions  $x \in C[I, E]$  for the nonlinear functional integral inclusion

$$x(t) \in a(t) + \int_0^t F(t, s, g(s, x(m(s))))ds, \quad t \in I = [0, T]$$

where  $E$  is reflexive Banach space and the set-valued function  $F$  satisfy Caratheodory condition.

**Key Words and Phrases:** Set-valued functions, weak solutions, functional integral inclusion, fixed point, Caratheodory condition.

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