

M-CONSTANTS IN ORLICZ-LORENTZ SEQUENCE SPACES WITH APPLICATIONS TO FIXED POINT THEORY

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Abstract. In this paper some estimates of M -constants in Orlicz-Lorentz sequence spaces for both, the Luxemburg and the Amemiya norms are given. Since degenerated Orlicz functions φ and degenerated weighted sequences ω are also admitted, this investigations concern the most possible wide class of Orlicz-Lorentz sequence spaces. M -constants were defined in 1969 by E.A. Lifshits, and used in the study of lattice structures on Banach spaces, as well as in the fixed point theory, by a number of authors. In the last section of the paper an application of our results to the fixed point property is presented.

Key Words and Phrases: M -constants, L -constants, lattice structures on Banach spaces, Riesz angle, fixed point property, symmetric spaces, Orlicz-Lorentz spaces, Orlicz functions, Luxemburg norm, Amemiya norm.

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