R_δ-STRUCTURE OF SOLUTIONS SET FOR A VECTOR EVOLUTION INCLUSIONS DEFINED ON RIGHT HALF-LINE

YI CHENG[*,1], RAVI P AGARWAL[**] AND SITIAN QIN[***]

[*]Department of Mathematics, Bohai University
Jinzhou, 121013, PR China
E-mail: chengyi407@yahoo.com

[**]Department of Mathematics, Texas A&M University-Kingsville
Kingsville, TX 78363, USA
E-mail: agarwal@tamuk.edu

[***]Department of Mathematics, Harbin Institute of Technology at Weihai
Weihai 264209, PR China
E-mail: qinsitian@163.com

Abstract. In this paper, we deal with the topological structure of a first order vector differential inclusion defined on right half-line. Under some general growth conditions, the R_δ structure of continue solution set for Cauchy problem on compact interval is investigated. Then by the inverse limit method, the R_δ structure is also obtained on noncompact interval. Further, using the related results of structure, we obtain the existence and topological structure of solution set for some nonlocal problems. Subsequently a optimal dual control problem is considered and an R_δ structure of attainable set based on the proven results is obtained.
Key Words and Phrases: Vector differential inclusion, topological structure, nonlocal condition, inverse limit, growth condition, R_δ set.
2010 Mathematics Subject Classification: 34B15, 34B16, 37J40.

REFERENCES

[1] V.N. Afanasiev, V.B. Kolmanovskii, V.R. Nosov, *Mathematical Theory of Control Systems Design*, Math. Appl., **341**, Kluwer Academic Publishers, Dordrecht, 1995.

[2] J. Andres, G. Gabor, L. Górniewicz, *Boundary value problems on infinite intervals*, Trans. Amer. Math. Soc., **351**(1999), 4861-4903.

[3] J. Andres, M. Pavlăcková, *Topological structure of solution sets to asymptotic boundary value problems*, J. Diff. Equa., **248**(2010), 127-150.

[4] A. Bakowska, G. Gabor, *Topological structure of solution sets to differential problems in Fréchet spaces*, Ann. Polon. Math., **95**(2009), no. 1, 17-36.

---

[1]Corresponding author.

[5] A. Bressan, G. Colombo, *Extensions and selection of maps with decomposable values*, Studia Math., **90**(1988), 69-86.

[6] H. Brezis, *Operateurs Maximaux Monotones*, North-Holland, Amsterdam, 1973.

[7] S.S. Ceron, O. Lopes, *α-Contractions and attractors for dissipative semilinear hyperbolic equations and systems*, Ann. Mat. Pura Appl., **160**(1991), 193-206.

[8] D.H. Chen, R.N. Wang, Y. Zhou, *Nonlinear evolution inclusions: Topological characterizations of solution sets and applications*, J. Funct. Anal., **265**(2013), no. 9, 2039-2073.

[9] F.S. DeBlasi, J. Myjak, *On the solutions sets for differential inclusions*, Bull. Pol. Acad. Sci. Math., **12**(1985), 17-23.

[10] F.S. De Blasi, G. Pianigiani, *Topological properties of nonconvex differential inclusions*, Nonlinear Anal., **20**(1993), 871-894.

[11] K. Deimling, *Multivalued Differential Equations*, De Gruyter, Berlin, 1992.

[12] M. Forti, P. Nistri, D. Papini, *Global exponential stability and global convergence in finite time of delayed neural networks with infinite gain*, IEEE Trans. Neural Networks, **16**(2005), 1449-1463.

[13] G. Gabor, *Acyclicity of solution sets of inclusions in metric spaces*, Topol. Meth. Nonlinear Anal., **14**(1999), 327-343.

[14] G. Gabor, *Some results on existence and structure of solution sets to differential inclusions on the half line*, Boll. Un. Mat. Ital. Sez. Bartic. Ric. Mat., **5**(2002), no. 8, 431-446.

[15] G. Gabor, A. Grudzka, *Structure of the solution set to impulsive functional differential inclusions on the half-line*, Nonlinear Diff. Eq. Appl., **19**(2012), 609-627.

[16] L. Górniewicz, *Topological Fixed Point Theory of Multivalued Mappings*, 2nd ed., Springer, Berlin, 2006.

[17] L. Górniewicz, M. Lassonde, *Approximation and fixed points for compositions of R_δ-maps*, Topology Appl., **55**(1994), no. 3, 239-250.

[18] S.C. Hu, N.S. Papageorgiou, *On the topological regularity of the solution set of differential inclusions with constraints*, J. Diff. Eq., **107**(1994), no. 2, 280-289.

[19] M. Kamenskii, V. Obukhovskii, P. Zecca, *Condensing Multivalued Maps and Semilinear Differential Inclusions in Banach Spaces*, de Gruyter Ser. Nonlinear Anal. Appl., vol.7, Walter de Gruyter, Berlin, New York, 2001.

[20] T.D. Ke, V. Obukhovskii, N.C. Wong, J.C. Yao, *Approximate control ability for system governed by nonlinear Volterra type equations*, Differ. Equ. Dyn. Syst., **20**(2012), no. 1, 35-52.

[21] S. Nadler, *Multivalued contraction mappings*, Pacific J. Math., **30**(1969), 475-488.

[22] V. Obukhovskii, P. Rubbioni, *On a control ability problem for systems governed by semilinear functional differential inclusions in Banach spaces*, Topol. Meth. Nonlinear Anal., **15**(2000), 141-151.

[23] D. O'Regan, *Topological structure of solution sets in Fréchet spaces: The projective limit approach*, J. Math. Anal. Appl., **324**(2006), 1370-1380.

[24] D. O'Regan, A. Petruşel, Leray-Schauder, *Lefschetz and Krasnoselskii fixed point theory in Fréchet spaces for general classes of Volterra operators*, Fixed Point Theory, **8**(2008), no. 2, 497-513.

[25] N.S. Papageorgiou, *Convergence theorems for Banach space valued integrable multifunctions*, Int. J. Math. Sci., **10**(1987), 433-442.

[26] B. Ricceri, *Une propriété topologique de l'ensemble des points fixes d'une contraction multivoque á valeurs convexes*, Atti Accad. Naz. Lincei Cl. Sci. Fis. Mat. Natur. Rend. Lincei, Mat. Appl., **81**(1987), no. 9, 283-286.

[27] V. Sĕda, Š. Belohorec, *A remark on second order functional differential systems*, Arch. Math., Brno, **36**(1993), no. 3-4, 169-176.

[28] V. Staicu, *On the solution sets to nonconvex differential inclusions of evolution type*, Discrete Contin. Dyn. Syst., **2**(1998), 244-252.

[29] I.I. Vrabie, *Compactness Methods for Nonlinear Evolutions*, Second Edition, Pitman Monogr. Surv. Pure Appl. Math., vol.75, Longman, John Wiley and Sons, 1995.

[30] Q.J. Zhu, *On the solution set of differential inclusions in Banach space*, J. Diff. Eq., **93**(1991), 213-237.