

## A STRONGLY CONVERGENT MODIFICATION OF THE PROXIMAL POINT ALGORITHM IN NONSMOOTH BANACH SPACES

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**Abstract.** Rockafellar's proximal point algorithm is known to be not strongly convergent in general in an infinite-dimensional Hilbert space. Effort has thus been made to modify this algorithm so that strong convergence is guaranteed. In this paper we provide a strongly convergent modification of Rockafellar's proximal point algorithm in a uniformly convex Banach space which is not necessarily smooth.

**Key Words and Phrases:** Maximal monotone operator, proximal point algorithm, strong convergence, generalized projection, uniformly convex Banach space, zero point.

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