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NONLINEAR PERTURBED INTEGRAL EQUATIONS RELATED TO NONLOCAL BOUNDARY VALUE PROBLEMS

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Abstract. By topological arguments, we prove new results on the existence, non-existence, localization and multiplicity of nontrivial solutions of a class of perturbed nonlinear integral equations. These type of integral equations arise, for example, when dealing with boundary value problems where nonlocal terms occur in the differential equation and/or in the boundary conditions. Some examples are given to illustrate the theoretical results.
Key Words and Phrases: Perturbed integral equation, nonlocal differential equation, nonlinear boundary condition, nontrivial solution, fixed point index, cone.
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References

[1] E. Alves, T.F. Ma, M.L. Pelicer, *Monotone positive solutions for a fourth order equation with nonlinear boundary conditions*, Nonlinear Anal., **71**(2009), 3834–3841.

[2] H. Amann, *Fixed point equations and nonlinear eigenvalue problems in ordered Banach spaces*, SIAM. Rev., **18**(1976), 620–709.

[3] D. Andrade, T.F. Ma, *Numerical solutions for a nonlocal equation with reflection of the argument*, Neural Parallel Sci. Comput., **10**(2002), 227–233.

[4] D. Bugajewski, M. Zima, *On the spectral radius of linearly bounded operators and existence results for functional-differential equations*, Proc. Fourth International Conf. on Dynamical Systems and Differential Equations, (2002), 147–155.

[5] A. Cabada, *An overview of the lower and upper solutions method with nonlinear boundary value conditions*, Bound. Value Probl. (2011), Art. ID 893753, 18 pp.

[6] A. Cabada, G. Infante, F.A.F. Tojo, *Nontrivial solutions of perturbed Hammerstein integral equations with reflections*, Bound. Value Probl., **86**(2013).

[7] A. Cabada, G. Infante, F.A.F. Tojo, *Nonzero solutions of perturbed Hammerstein integral equations with deviated arguments and applications*, Topol. Methods Nonlinear Anal., to appear.

[8] M. Chipot, J.F. Rodrigues, *On a class of nonlinear nonlocal elliptic problems*, Math. Model. Num. Anal., **26**(1992), 447–467.

[9] R. Conti, *Recent trends in the theory of boundary value problems for ordinary differential equations*, Boll. Un. Mat. Ital., **22**(1967), 135–178.

[10] K. Deimling, *Nonlinear Functional Analysis*, Springer-Verlag, Berlin, 1985. Reprinted, Dover Publications.

[11] V. Dolezal, *Some results on the invertibility of nonlinear operators*, Circuits Systems Signal Process, **17-6**(1998), 683–690.

[12] V. Dolezal, *The invertibility of operators and contraction mappings*, Circuits Systems Signal Process, **18-2**(1999), 183–187.

[13] H. Fan, R. Ma, *Loss of positivity in a nonlinear second order ordinary differential equations*, Nonlinear Anal., **71**(2009), 437–444.

[14] R. Figueroa, R.L. Pouso, *Minimal and maximal solutions to second-order boundary value problems with state-dependent deviating arguments*, Bull. London Math. Soc., **43**(2011), 164–174.

[15] D. Franco, D. O'Regan, J. Perán, *Fourth-order problems with nonlinear boundary conditions*, J. Comput. Appl. Math., **174**(2005), 315–327.

[16] D. Franco, G. Infante, D. O'Regan, *Positive and nontrivial solutions for the Urysohn integral equation*, Acta Math. Sinica, **22**(2006), 1745–1750.

[17] C.S. Goodrich, *On nonlocal BVPs with nonlinear boundary conditions with asymptotically sublinear or superlinear growth*, Math. Nachr., **285**(2012), 1404–1421.

[18] C.S. Goodrich, *Positive solutions to boundary value problems with nonlinear boundary conditions*, Nonlinear Anal., **75**(2012), 417–432.

[19] C.S. Goodrich, *On nonlinear boundary conditions satisfying certain asymptotic behavior*, Nonlinear Anal., **76**(2013), 58–67.

[20] C.S. Goodrich, *A note on semipositone boundary value problems with nonlocal, nonlinear boundary conditions*, Arch. Math. (Basel), **103**(2014), 177–187.

[21] C.S. Goodrich, *Semipositone boundary value problems with nonlocal, nonlinear boundary conditions*, Adv. Differential Equations, **20**(2015), 117–142.

[22] P. Guidotti, S. Merino, *Gradual loss of positivity and hidden invariant cones in a scalar heat equation*, Differential Integral Equations, **13**(2000), 1551–1568.

[23] D. Guo, V. Lakshmikantham, *Nonlinear Problems in Abstract Cones*, Academic Press, Boston, 1988.

[24] G. Infante, *Eigenvalues of some non-local boundary-value problems*, Proc. Edinb. Math. Soc., **46**(2003), 75–86.

[25] G. Infante, *Nonzero solutions of second order problems subject to nonlinear BCs*, Dynamic systems and applications, Vol. 5, 222–226, Dynamic, Atlanta, GA, 2008.

[26] G. Infante, *Nonlocal boundary value problems with two nonlinear boundary conditions*, Commun. Appl. Anal., **12**(2008), 279–288.
[27] G. Infante, *Positive solutions of a doubly nonlocal boundary value problem*, Commun. Appl. Anal., **18**(2014), 147–154.
[28] G. Infante, P. Pietramala, *Perturbed Hammerstein integral inclusions with solutions that change sign*, Comment. Math. Univ. Carolin., **50**(2009), 591–605.
[29] G. Infante, P. Pietramala, *A cantilever equation with nonlinear boundary conditions* Electron. J. Qual. Theory Differ. Equ., Spec. Ed. I, **15**(2009), 1–14.
[30] G. Infante, P. Pietramala, F.A.F. Tojo, *Nontrivial solutions of local and nonlocal Neumann boundary value problems*, Proc. Roy. Soc. Edinburgh Sect. A, to appear.
[31] G. Infante, J.R.L. Webb, *Nonzero solutions of Hammerstein integral equations with discontinuous kernels*, J. Math. Anal. Appl., **272**(2002), 30–42.
[32] G. Infante, J.R.L. Webb, *Three point boundary value problems with solutions that change sign*, J. Integral Equations Appl., **15**(2003), 37–57.
[33] G. Infante, J.R.L. Webb, *Nonlinear nonlocal boundary value problems and perturbed Hammerstein integral equations*, Proc. Edinb. Math. Soc., **49**(2006), 637–656.
[34] T. Jankowski, *Solvability of three point boundary value problems for second order differential equations with deviating arguments*, J. Math. Anal. Appl., **312**(2005), 620–636.
[35] T. Jankowski, *Multiple solutions for a class of boundary-value problems with deviating arguments and integral boundary conditions*, Dynam. Systems Appl., **19**(2010), 179–188.
[36] T. Jankowski, *Positive solutions to second-order differential equations with dependence on the first-order derivative and nonlocal boundary conditions*, Bound. Value Probl., **8**(2013).
[37] G.L. Karakostas, *Existence of solutions for an n-dimensional operator equation and applications to BVPs*, Electron. J. Differential Equations, (2014), no. 71, 17 pp.
[38] G.L. Karakostas, P. Ch. Tsamatos, *Existence of multiple positive solutions for a nonlocal boundary value problem*, Topol. Meth. Nonlinear Anal., **19**(2002), 109–121.
[39] G.L. Karakostas, P. Ch. Tsamatos, *Multiple positive solutions of some Fredholm integral equations arisen from nonlocal boundary-value problems*, Electron. J. Differential Equations, (2002), no. 30, 17 pp.
[40] M.A. Krasnosel'skiĭ, P.P. Zabreĭko, *Geometrical Methods of Nonlinear Analysis*, Springer-Verlag, Berlin, 1984.
[41] K.Q. Lan, *Multiple positive solutions of Hammerstein integral equations with singularities*, Differential Equations and Dynamical Systems, **8**(2000), 175–195.
[42] R. Ma, *A survey on nonlocal boundary value problems*, Appl. Math. E-Notes, **7**(2001), 257–279.
[43] J.J. Nieto, J. Pimentel, *Positive solutions of a fractional thermostat model*, Bound. Value Probl., 2013:**5**(2013).
[44] S.K. Ntouyas, *Nonlocal initial and boundary value problems: a survey*, Handbook of Differential Equations: Ordinary Differential Equations, Vol. II, 461–557, Elsevier B. V., Amsterdam, 2005.
[45] D. Piao, *Pseudo almost periodic solutions for differential equations involving reflection of the argument*, J. Korean Math. Soc., **41**(2004), 747–754.
[46] D. Piao, *Periodic and almost periodic solutions for differential equations with reflection of the argument*, Nonlinear Anal., **57**(2004), 633–637.
[47] D. Piao, Na Xin, *Bounded and almost periodic solutions for second order differential equation involving reflection of the argument*, arXiv:1302.0616.
[48] M. Picone, *Su un problema al contorno nelle equazioni differenziali lineari ordinarie del secondo ordine*, Ann. Scuola Norm. Sup. Pisa Cl. Sci., **10**(1908), 1–95.
[49] P. Pietramala, *A note on a beam equation with nonlinear boundary conditions*, Bound. Value Probl., (2011), Art. ID 376782, 14 pp.
[50] A. Štikonas, *A survey on stationary problems, Green's functions and spectrum of Sturm-Liouville problem with nonlocal boundary conditions*, Nonlinear Anal. Model. Control, **19**(2014), 301–334.
[51] W. Szatanik, *Quasi-solutions for generalized second order differential equations with deviating arguments*, J. Comput. Appl. Math., **216**(2008), 425–434.

[52] W. Szatanik, *Minimal and maximal solutions for integral boundary value problems for the second order differential equations with deviating arguments*, Dynam. Systems Appl., **19**(2010), 87–96.
[53] J.R.L. Webb, *Solutions of nonlinear equations in cones and positive linear operators*, J. London Math. Soc., **82**(2010), 420–436.
[54] J.R.L. Webb, *A class of positive linear operators and applications to nonlinear boundary value problems*, Topol. Meth. Nonlinear Anal., **39**(2012), 221–242.
[55] J.R.L. Webb, *Existence of positive solutions for a thermostat model*, Nonlinear Anal. Real World Appl., **13**(2012), 923–938.
[56] J.R.L. Webb, G. Infante, *Positive solutions of nonlocal boundary value problems: a unified approach*, J. London Math. Soc., **74**(2006), 673–693.
[57] J.R.L. Webb, G. Infante, *Positive solutions of nonlocal boundary value problems involving integral conditions*, NoDEA - Nonlinear Differential Equations Appl., **15**(2008), 45–67.
[58] J.R.L. Webb, G. Infante, *Nonlocal boundary value problems of arbitrary order*, J. London Math. Soc., **79**(2009), 238–258.
[59] J.R.L. Webb, K.Q. Lan, *Eigenvalue criteria for existence of multiple positive solutions of nonlinear boundary value problems of local and nonlocal type*, Topol. Meth. Nonlinear Anal., **27**(2006), 91–115.
[60] W.M. Whyburn, *Differential equations with general boundary conditions*, Bull. Amer. Math. Soc., **48**(1942), 692–704.
[61] J. Wiener, A.R. Aftabizadeh, *Boundary value problems for differential equations with reflection of the argument*, Internat. J. Math. Math. Sci., **8**(1985), 151–163.
[62] Z. Yang, *Positive solutions to a system of second-order nonlocal boundary value problems*, Nonlinear Anal., **62**(2005), 1251–1265.
[63] Z. Yang, *Positive solutions of a second-order integral boundary value problem*, J. Math. Anal. Appl., **321**(2006), 751–765.
[64] M. Zima, *Applications of the spectral radius to some integral equations*, Comment. Math. Univ. Carolin., **36**(1995), 695–703.

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