

Δ -CONVERGENCE THEOREMS FOR INVERSE-STRONGLY MONOTONE MAPPINGS IN CAT(0) SPACES

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Abstract. In this paper, we first define and study inverse-strongly monotone mappings in general metric spaces. Then, we prove the existence theorem of solutions for variational inequalities involving such mappings. Finally, we introduce an iterative process for finding a common element of the set of fixed points of a nonexpansive mapping and the set of solutions of a variational inequality problem for inverse-strongly monotone mappings in CAT(0) metric spaces.

Key Words and Phrases: inverse-strongly monotone mapping; nonexpansive mapping; variational inequality; fixed point; CAT(0) metric space.

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