

AMENABLE LOCALLY COMPACT SEMIGROUPS AND A FIXED POINT PROPERTY

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Abstract. For a locally compact semigroup S , we study a general fixed point property in terms of Banach left S -modules. We then use this property to give our main result which is a new characterization for left amenability of a large class of locally compact semigroups; finally, we investigate several examples which lead us to the conjecture that the main result remains true for all locally compact semigroups.

Key Words and Phrases: Banach left S -module, foundation semigroup, left amenability, left fixed point, left invariant mean, locally compact semigroup, weak*-operator topology.

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