

CONTROLLABILITY RESULTS FOR FRACTIONAL ORDER NEUTRAL FUNCTIONAL DIFFERENTIAL INCLUSIONS WITH INFINITE DELAY

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Abstract. In this paper, we investigate the controllability results for fractional order neutral functional differential inclusions with an infinite delay involving the Caputo derivative in Banach spaces. First, we establish a set of sufficient conditions for the controllability of fractional order neutral functional differential inclusions with infinite delay in Banach spaces. The main techniques rely on Bohnenblust-Karlin's fixed point theorem, operator semigroups and fractional calculus. Further, we extend this result to study the controllability concept with nonlocal conditions. An example is also given to illustrate our main results.

Key Words and Phrases: Controllability, fractional integro-differential inclusions, neutral equations, semigroup theory, multivalued map, Bohnenblust-Karlin's fixed point theorem.

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