

SATURATED FIBRE CONTRACTION PRINCIPLE

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Abstract. For a triangular operator $A : X \times Y \rightarrow X \times Y$, $A = (B, C)$, where $B : X \rightarrow X$ and $C : X \times Y \rightarrow Y$ we study in which conditions on operators $B : X \rightarrow X$ and $C : X \times Y \rightarrow Y$ we have that:

- (1) the fixed point problem for triangular operator $A = (B, C)$ is well posed
- (2) the operator $A = (B, C)$ has the Ostrowski property
- (3) the fixed point equation $(x, y) = A(x, y)$ is generalized Ulam-Hyers stable.

Key Words and Phrases: Cauchy lemma, fixed point, fibre contraction principle, well-posedness of the fixed point problem, Ostrowski property, Ulam-Hyers stability, generalized Ulam-Hyers stability.

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