Fixed Point Theory, 18(2017), No. 2, 709-720
DOI 10.24193/fpt-ro.2017.2.57
http://www.math.ubbcluj.ro/~nodeacj/sfptcj.html

COMMON FIXED POINT THEOREMS FOR GENERALIZED NON-EXPANSIVE SEMI-TOPOLOGICAL SEMIGROUPS IN LOCALLY CONVEX SPACES

TAMER NABIL[1,2] AND AHMED H. SOLIMAN[1,3]

[1]King Khalid University, Faculty of Science, Mathematics Department, Abha 9004, Saudi Arabia
[2]Suez Canal University, Faculty of Computers and Informatics
Department of Basic Science, Ismailia, Egypt
E-mail: $t_3$bdelsadek@yahoo.com

[3]Department of Mathematics, Faculty of Science, Al-Azhar University
Assiut Branch, Assiut 71524, Egypt
E-mail: ahsolimanm@gmail.com

Abstract. In this paper, we shall be concerned with a special kind of equicontinuous semi-topological semigroups of self-mappings on a weakly compact convex subset of a separated locally convex space, namely, the generalized non-expansive mappings and we shall introduce some common fixed point results for this kind of semigroups. Also, we study a characterization of the existence of a left invariant mean on almost and weakly almost periodic functions on separable semi-topological semigroups. Our results extend the results due to Lau and Zhang [17] and Lau [13].
Key Words and Phrases: Fixed point property, non-expansive mapping, generalized non-expansive mapping, weakly compact convex set, weakly almost periodic, reversible semigroup, invariant mean.
2010 Mathematics Subject Classification: 47H10, 47H20, 43A07.

Acknowledgments. The authors extend their appreciation to the Deanship of Scientific Research at King Khalid University for funding this work through General Research Project under grant number (G.R.P- 009-38). Also, the authors thank Prof. Dr. Anthony To-Ming Lau (University of Alberta, Edmonton, Alberta, Canada) with whom I had fruitful discussions and useful comments regarding this work. And, special thanks are due to the referees for their deep insight.

References

[1] A. Abkar, M. Eslamian, *Fixed point theorems for Suzuki generalized nonexpansive multivalued mappings in Banach spaces*, Fixed Point Theory Appl., **2010**(2010), Article ID 457935.
[2] T.D. Benavides, M.A.J. Pineda, *Fixed points of nonexpansive mappings in spaces of continuous functions*, Proc. Amer. Math. Soc., **133**(2005), 3037-3046.
[3] T.D. Benavides, M.A.J. Pineda, S. Prus, *Weak compactness and fixed point property for affine mappings*, J. Funct. Anal., **209**(2004), 1-15.
[4] F.E. Browder, *Non-expansive nonlinear operators in Banach spaces*, Proc. Natl. Acad. Sci. USA, **54**(1965), 1041-1044.

[5] A.H. Clifford, G.B. Preston, *The Algebraic Theory of Semigroups*, Vol. 1, Math. Surveys, no. 7, Amer. Math. Soc., Providence, R.I., 1961.
[6] M.M. Day, *Amenahle semigroups*, Illinois J. Math., **1**(1957), 509-544.
[7] R.E. De Marr, *Common fixed points for commuting contraction mappings*, Pacific J. Math., **13**(1963), 1139-1141.
[8] P.N. Dowling, C.J. Lennard, B. Turett, *Weak compactness is equivalent to the fixed point property in c0*, Proc. Amer. Math. Soc., **132**(2004), 1659-1666.
[9] K. Goebel, W.A. Kirk, *Topics in Metric Fixed Point Theory*, Cambridge Stud. Adv. Math., vol. 28, Cambridge Univ. Press, Cambridge, 1990.
[10] K. Goebel, W.A. Kirk, *Classical theory of nonexpansive mappings*, in: Handbook of Metric Fixed Point Theory, Kluwer Acad. Publ., Dordrecht, 2001, 49–91.
[11] R.D. Holmes, A. Lau, *Nonexpansive actions of topological semigroups and fixed points*, J. London Math. Soc., **5**(1972), 330-336.
[12] W.A. Kirk, *A fixed point theorem for mappings which do not increase distances*, Amer. Math. Monthly, **72**(1965), 1004-1006.
[13] A.T.-M. Lau, *Invariant means on almost periodic functions and fixed point properties*, Rocky Mountain J. Math., **3**(1973), 69-76.
[14] A.T.-M. Lau, *Some fixed point theorems and W^*-algebras*, Fixed Point Theory and Applications (S. Swaminathan-Ed.), Academic Press, (1976), 121-129.
[15] A.T.-M. Lau, *Amenability of semigroups, The analytic and topological theory of semigroups* (K.H. Hoffmann, J.D. Lawson, J.S. Pym-Eds.), W. de Gruyter, Berlin, 1990, 313-334.
[16] A.T.-M. Lau, *Amenability and fixed point property for semigroup of nonexpansive mappings*, Fixed Point Theory and Applications (in: M.A. Thera, J.B. Baillon), Pitman Research Notes Mathematical Series, **252**(1991), 303-313.
[17] A.T.-M. Lau, Y. Zhang, *Fixed point properties of semigroups of non-expansive mappings*, J. Funct. Anal., **254**(2008), 2534-2554.
[18] A.T.-M. Lau, Y. Zhang, *Fixed point properties for semigroups of nonlinear mappings and amenability*, J. Funct. Anal., **263**(2012), no. 10, 2949 - 2977.
[19] T. Mitchell, *Fixed points of reversible semigroups of nonexpansive mappings*, Ködai Math. Sem. Rep., **22**(1970), 322-323.
[20] D.R. Sahu, D. O'Regan, R.P. Agarwal, *Fixed Point Theory for Lipschitzian-Type Mappings with Applications*, Springer-Verlag, Berlin, 2009.
[21] A.H. Soliman, *A coupled fixed point theorem for nonexpansive one parameter semigroup*, J. Adv. Math. Stud., **7**(2014), no. 2, 28-37.
[22] T. Suzuki, *Fixed point theorems and convergence theorems for some generalized nonexpansive mappings*, J. Math. Anal. Appl., **340**(2008), 1088-1095.
[23] W. Takahashi, *Fixed point theorem for amenable semigroups of nonexpansive mappings*, Kodai Math. Sem. Rep., **21**(1969), 383-386.

Received: November 5, 2014; Accepted: May 14, 2015.