A PROOF OF THE MARKOV-KAKUTANI THEOREM ON NONCOMPACT SET VIA ZERMELO'S WELL-ORDERING THEOREM

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Abstract. The Markov-Kakutani fixed point theorem has been considered as one of the most remarkable theorems due to considerable diversity in its applications in the history of functional analysis. Different approaches have been investigated to prove this theorem; however, the condition of compactness of the underlying set is essentially used. In this paper, we develop a new method, based on Zermelo's well-ordering theorem, to weaken the compactness condition.

Key Words and Phrases: Affine mapping, the Markov-Kakutani fixed point theorem, Zermelo's well-ordering theorem.

2010 Mathematics Subject Classification: 47H20, 47H10.

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