

REMARKS ON PASICKI'S ABSTRACT METRIC SPACES AND LIFSHITS'S CONSTANT

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Abstract. In attempting to formulate a more axiomatic approach to metric fixed point theory, some recent concepts due to Lech Pasicki give rise to a number of intriguing problems, especially regarding the Lifshits constant of such spaces. We review these results here, with an emphasis on a number of questions that seem to remain open. For the convenience of the reader certain proofs of known results are also included. Some new observations are also included.

Key Words and Phrases: Nonexpansive mappings, fixed points, bead spaces, disc spaces, Lifshitz constant.

2010 Mathematics Subject Classification: 54H25, 47H09, 47H10.

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Received: May 11, 2015; Accepted: July 10, 2015.

