

## ON THE HYPERSTABILITY OF $(m, n)$ -DERIVATIONS

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**Abstract.** Let  $\mathcal{A}$  be a unital algebra, let  $\mathcal{X}$  be a unital  $\mathcal{A}$ -module for which  $\mathcal{X}_\rho$  is a  $\rho$ -complete modular space and let  $f : \mathcal{A} \rightarrow \mathcal{X}_\rho$  be a mapping. We present some observations concerning hyperstability of the following functional equations

$$\mu f\left(\frac{x+y}{2}\right) + \mu f\left(\frac{x-y}{2}\right) = f(\mu x), \quad (m+n)f(xy) = 2mx \cdot f(y) + 2ny \cdot f(x)$$

for all  $x, y \in \mathcal{A}$  and all  $\mu \in \mathbb{T}_{1/n_0} = \{e^{i\theta}; 0 \leq \theta \leq 2\pi/n_0\}$ , where  $m, n \geq 0$  with  $m+n \neq 0$  are fixed integers.

**Key Words and Phrases:** Approximately  $(m, n)$ -derivation, fixed point, hyperstability, modular space, unital algebra.

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