

## FIXED POINTS OF LOCALLY FUZZY CONTRACTIVE SET-VALUED MAPPINGS IN FUZZY METRIC SPACES

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**Abstract.** In this paper, we first introduce new notions of the locally fuzzy contraction of set-valued mappings and  $\varepsilon$ -chainable fuzzy metric space. Using these notions, we deal with some issues of fixed point theory involving the generalization of fuzzy contractive mappings introduced by other authors. We enlarge this class and establish fuzzy versions of some known fixed point theorems (such as the Nadler's set-valued contractive and Edelstein's locally contractive fixed point theorems). The results are supported by examples.

**Key Words and Phrases:** Locally fuzzy contractive mapping,  $\varepsilon$ -chainable fuzzy metric space,  $fw$ -distance, fixed point.

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