BOUNDARY VALUE PROBLEM FOR FRACTIONAL DIFFERENTIAL EQUATIONS WITH NONLOCAL CONDITIONS IN BANACH SPACES

SAMIRA HAMANI*, JOHNNY HENDERSON** AND JUAN J. NIETO ***

*Department of Mathematics, University of Mostaganem
B.P. 227, 27000, Mostaganem, Algeria
E-mail: hamani_samira@yahoo.fr

**Department of Mathematics, Baylor University
Waco, TX 76798-7328, USA
E-mail: johnny_Henderson@baylor.edu

***Faculty of Mathematics, University de Santiago de Compostela
15782 Santiago de Compostela, Spain
and
Faculty of Science, King Abdulaziz University
P.O. Box 80203, Jeddah 21585, Saudi Arabia
E-mail: juanjose.nieto.roig@usc.es

Abstract. In this paper, we investigate the existence of solution for boundary value problem with fractional differential equations with nonlocal conditions involving the standard Riemann-Liouville fractional derivative of order $r \in (1, 2]$.

Key Words and Phrases: Differential equation, Riemann-Liouville fractional derivative, fractional integral, nonlocal condition, existence, fixed point, Banach space.

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