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## ON ORTHOGONAL SETS AND BANACH FIXED POINT THEOREM

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**Abstract.** We introduce the notion of the orthogonal sets and give a real generalization of Banach' fixed point theorem. As an application, we find the existence of solution for a first-order ordinary differential equation.

Key Words and Phrases: Orthogonal set, fixed point, differential equation, Picard operator. 2010 Mathematics Subject Classification: 47H10, 54H25.

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## References

- [1] J. Diestel, Geometry of Banach Spaces, Springer-Verlag, Berlin-Heidelberg-New York, 1975.
- [2] C.R. Diminnie, A new orthogonality relation for normed linear spaces, Math. Nachr., 114(1983), 197–203.
- [3] A.C.M. Ran, M.C.B. Reurings, A fixed point theorem in partially ordered sets and some applications to matrix equations, Proc. Amer. Math. Soc., 132(2004), 1435–1443.

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