FIXED POINT THEOREMS FOR SET-VALUED MAPPINGS AND VARIATIONAL PRINCIPLES IN UNIFORM SPACES WITH w-DISTANCES

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Abstract. By making use of w-distances, for set-valued mappings defined on uniform spaces, we generalize some classical results in the existing literature of nonlinear analysis, such as, Bishop-Phelps and Caristi's fixed point theorems, Ekeland's ϵ-variational principle and the nonconvex minimization theorem according to Takahashi. Our version of Caristi's fixed point theorem is used to prove existence of fixed points on uniform spaces for some contractions such as weak, Chatterjea and Kannan contractions defined by means of w-distances. The results introduced in this paper generalize others existing in the literature of nonlinear analysis.

Key Words and Phrases: Caristi's theorem, fixed point, uniform spaces, variational principle, w-distance.

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