

A CONTRACTION PRINCIPLE ON GAUGE SPACES WITH GRAPHS AND APPLICATION TO INFINITE GRAPH-DIRECTED ITERATED FUNCTION SYSTEMS

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Abstract. We consider multi-valued maps defined on a complete gauge space endowed with a directed graph. We establish a fixed point result for maps which send connected points into connected points and satisfy a generalized contraction condition. Then, we study infinite graph-directed iterated function systems (H -IIFS). We give conditions insuring the existence of a unique attractor to an H -IIFS. Finally, we apply our fixed point result for multi-valued contractions on gauge spaces endowed with a graph to obtain more information on the attractor of an H -IIFS. More precisely, we construct a suitable gauge space endowed with a graph G and a suitable multi-valued G -contraction such that its fixed points are sub-attractors of the H -IIFS.

Key Words and Phrases: Fixed point, multi-valued map, contraction, graph, graph-directed iterated function system, infinite system, attractor gauge space.

2010 Mathematics Subject Classification: 47H10, 47H04, 47H09, 28A80, 54E15.

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Received: April 23, 2014; Accepted: January 23, 2016.