

**APPROXIMATING FIXED POINT SOLUTIONS
OF VARIATIONAL INEQUALITIES USING EXPLICIT
ITERATIONS FOR ASYMPTOTICALLY NONEXPANSIVE
SEMIGROUP OF MAPPINGS IN BANACH SPACES**

R.C. DIMRI AND PUSHPENDRA SEMWAL*

*Department of Mathematics, H.N.B. Garhwal University
Srinagar Garhwal, India-246174
E-mail: psrtdm@gmail.com

Abstract. Our purpose in this article is to introduce two new iterations and studying the strong convergence algorithms for finding approximating solutions of some variational inequalities on the set of common fixed point for a semigroup of asymptotically nonexpansive mappings. The results of this article improve and extend the recent work of Sunthrayuth and Kumam [21].

Key Words and Phrases: Common fixed point, variational inequality, semigroup of asymptotically nonexpansive mapping, strong convergence.

2010 Mathematics Subject Classification: 47H09, 47H10, 47H20, 47J20, 54H25.

REFERENCES

- [1] I.A. Aleyner, Y.Censor, *Best approximation to common fixed points of a semigroup of nonexpansive operators*, J. Nonlinear Convex Anal., **6**(2005), 137-151.
- [2] W.L. Bynum, *Normal structure coefficients for Banach spaces*, Pacific J. Math., **86**(2001), 427-436.
- [3] C. Byrne, *A unified treatment of some iterative algorithms in signal processing and image reconstruction*, Inverse Probl., **20**(2004), 103-120.
- [4] F.E. Browder, *Convergence theorems for sequences of nonlinear operators in Banach Spaces*, Math. Z., **100**(1967), 201-225.
- [5] P. Cholamjiak, S. Suantai, *Viscosity approximation methods for a nonexpansive semigroup in Banach spaces with gauge functions*, J. Global Optim., **54**(2012), 185-197.
- [6] R. Chen, Y. Song, *Convergence to common fixed point of nonexpansive semigroup*, J. Comput. Appl. Math., **200**(2007), 500-575.
- [7] G. Cai, C.S. Hu, *Strong convergence theorems of a general iterative process for a finite family of λ -strict pseudo-contraction in q -uniformly smooth Banach spaces*, Comput. Math. Appl., **59**(2010), 49-160.
- [8] F. Deutsch, I. Yamada, *Minimizing certain convex functions over the intersection of the fixed points sets of nonexpansive mappings*, Numer. Funct. Anal. Optim., **19**(1998), 33-56.

- [9] K. Goebel, W.A. Kirk, *A fixed point theorem for asymptotically nonexpansive mappings*, Proc. Amer. Math.Soc., **35**(1972), 171-174.
- [10] A.T.-M. Lau, *Amenability and fixed point property for semigroup of nonexpansive mapping*, In: Thera, M.A., Baillon, J.B. (eds), Fixed Point Theory and Application, Pitman Res. Notes Math., Set, Zongman, Harlow, **252**(1991), 303-313.
- [11] A.T.-M. Lau, *Invariant means and fixed point properties of semigroup of nonexpansive mappings*, Taiwanese J. Math., **12**(2008), 1525-1542.
- [12] A.T.-M. Lau, H. Miyake, W. Takahashi, *Approximation of fixed points for amenable semigroups of nonexpansive mappings in Banach spaces*, Nonlinear Anal., **67**(2007), 1211-1225.
- [13] S. Li, L. Li, Y. Su, *General iterative methods for one-parameter nonexpansive semigroup in Hilbert space*, Nonlinear Anal., **70**(2009), 3065-3071.
- [14] T.C. Lim, H.K. Xu, *Fixed point theorems for asymptotically nonexpansive mappings*, Nonlinear Anal., **22**(1994), 1345-1355.
- [15] A. Moudafi, *Viscosity approximation methods for fixed point problems*, J. Math. Anal. Appl., **241**(2000), 46-55.
- [16] G. Marino, H.K. Xu, *A general iterative for nonexpansive mapping in Hilbert spaces*, J. Math. Anal. Appl., **318**(2006), 43-52.
- [17] C.I. Podilchuk, R.J. Mammone, *Image recovery by convex projections using a least squares constraint*, J. Opt. Soc. Am., **A7**(2005), 517-521.
- [18] S. Reich, D. Shoiklet, *Nonlinear Semigroups, Fixed Points and Geometry of Domains in Banach Spaces*, Imperial Collage Press, London, 2005.
- [19] M.I. Sezan, H. Stark, *Applications of convex projection theory to image recovery in tomography and related areas*, In: Stark, H. (ed.), Image Recovery Theory and Applications, Academic Press, Orlando, (1987), 415-462.
- [20] P. Sunthrayuth, P. Kumam, *Approximating solutions of variational inequalities on the sets of common fixed points for a semigroup of asymptotically nonexpansive mappings in Banach spaces*, Math. Sci., **6**(20)(2012).
- [21] P. Sunthrayuth, P. Kumam, *Fixed point solutions of variational inequalities for a semigroup of asymptotically nonexpansive mappings in Banach spaces*, Fixed Point Theory Appl., **177**(2012).
- [22] T. Suzuki, *Strong convergence of Krasnoselski and Mann's type sequence for one-parameter nonexpansive semigroup without Bochner integrals*, J. Math. Anal. Appl., **305**(2005), 227-239.
- [23] W. Takahashi, *Nonlinear Functional Analysis*, Yokahama Publishers Yokahama , 2000.
- [24] R. Wangkeeree, N. Petrot, R. Wangkeeree, *The general iterative methods of nonexpansive mappings in Banach spaces*, J. Global Optim., **51**(2011), 27-46.
- [25] H.K. Xu, *An iterative approach to quadratic optimization*, J. Optim. Theory Appl., **116**(2003), 659-678.
- [26] H.K. Xu, *Viscosity approximation methods for nonexpansive mappings*, J. Math. Anal. Appl., **298**(2004), 279-291.
- [27] H.K. Xu, *Iterative algorithms for nonlinear operators*, J. London Math. Soc., **66**(2002), no. 1, 240-256.
- [28] D. Youla, *Mathematical theory of image restoration by the method of convex projections*, In: Stark, H. (ed.), Image Recovery Theory and Applications, Academic Press, Orlando, 1987, 29-77.
- [29] D. Youla, *On deterministic convergence of iterations of relaxed projection operators*, J. Vis. Commun. Image Represent., **1**(1990), 12-20.
- [30] H. Zegeye, N. Shahzad, O.A. Daman, *Strong convergence theorems for a semigroup of asymptotically nonexpansive mappings*, Math. Comput. Model., **54**(2011), 2077-2086.
- [31] H. Zegeye, N. Shahzad, *Convergence theorems for strongly continuous semi-groups of asymptotically nonexpansive mappings*, Nonlinear Anal., **71**(2009), 2308-2315.

- [32] H. Zegeye, N. Shahzad, *Strong convergence theorems for continuous semigroups of asymptotically nonexpansive mappings*, Numer. Funct, Anal. Optim., **30**(2009), no. 7, 833-848.

Received: March 12, 2015; Accepted: May 20, 2015.