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ON PROPERTIES OF CONTRACTIONS AND NONEXPANSIVE MAPPINGS ON SPHERICAL CAPS IN HILBERT SPACES

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Abstract. Let H be an at least two-dimensional real Hilbert space with the unit sphere S_H. For $\alpha \in [-1,1]$ and $n \in S_H$ we define an (α,n)-spherical cap by $S_{\alpha,n} = \{x \in S_H : \langle x,n\rangle \geq \alpha\}$. We show that the distance between the set of contractions $T : S_{\alpha,n} \to S_{\alpha,n}$ and the identity mapping is positive iff $\alpha < 0$. We also study the fixed point property and the minimal displacement problem in this setting for nonexpansive mappings.
Key Words and Phrases: Contractions, nonexpansive mappings, fixed point property, almost fixed point property, minimal displacement.
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