

A NONLOCAL PROBLEM AT INFINITY FOR SECOND ORDER DIFFERENTIAL EQUATIONS

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Abstract. In this paper we propose the study of a scalar integral equation of the type

$$y(t) = g(y) + \int_t^\infty (s-t)a(s)f(y(s)) ds, \quad t \geq 0,$$

and give conditions on g , a and f that ensure the existence of solutions on $[0, \infty)$ which are asymptotically equal to $g(y)$ at ∞ . As a consequence, we obtain results on the existence of solutions for a problem of the type

$$y''(t) = a(t)f(y(t)), \quad y(\infty) = g(y),$$

where $y(\infty) = \lim_{t \rightarrow \infty} y(t)$. This problem could be thought as a sort of nonlocal problem at ∞ , and our conditions on f include the case of a linear equation.

Key Words and Phrases: Nonlocal problem, asymptotic behavior, integral equation, second order differential equation, Leray-Schauder type fixed point theorem.

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