

## COMPARABLE LINEAR CONTRACTIONS IN ORDERED METRIC SPACES

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**Abstract.** In this paper, with a view to improve the  $g$ -monotonicity condition, we introduce the notion of  $g$ -comparability of a mapping defined on an ordered set and utilize the same to prove some existence and uniqueness results on coincidence points for linear contraction without  $g$ -monotonicity in ordered metric spaces. Our results extend some classical and well known results due to Ran and Reurings (Proc. Amer. Math. Soc. **132**(2004), no.5, 1435-1443), Nieto and Rodríguez-López (Acta Math. Sin. **23**(2007), no.12, 2205-2212), Turinici (Libertas Math. **31**(2011), 49-55), Turinici (Math. Student **81**(2012), no.1-4, 219-229) and Dorić *et al.* (RACSAM **108**(2014), no.2, 503-510) and similar others.

**Key Words and Phrases:** Ordered metric space,  $g$ -monotone mappings, comparable mappings,  $TCC$  property, termwise monotone sequence.

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