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ASYMPTOTICALLY QUASI-NONEXPANSIVE MAPPINGS WITH RESPECT TO THE BREGMAN DISTANCE IN THE INTERMEDIATE SENSE

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Abstract. The purpose of this paper is to introduce a new class of nonlinear mappings which is an extension of asymptotically quasi-nonexpansive mappings with respect to the Bregman distance in the intermediate sense. A strong convergence theorem of the shrinking projection method with the modified Mann iteration is established to find fixed points of the mappings in reflexive Banach spaces.

Key Words and Phrases: Bregman distance, Bregman projection, asymptotically quasi-nonexpansive in the intermediate sense, fixed point, Legendre function, totally convex function.

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References

[1] Y.I. Alber, *Metric and generalized projection operators in Banach spaces: properties and applications*, Theory and Applications of Nonlinear Operators of Accretive and Monotone Type, Lecture Notes Pure Appl. Math., **178**, Dekker, New York, 1996, 15-50.

[2] V. Barbu, *Nonlinear Differential Equations of Monotone Types in Banach Spaces*, Springer, New York, 2010.

[3] H.H. Bauschke, J.M. Borwein, P.L. Combettes, *Essential smoothness, essential strict convexity, and Legendre functions in Banach spaces*, Commun. Contemp. Math., **3**(2001), no. 4, 615-647.

[4] H.H. Bauschke, M.S. Macklem, X. Wang, *Chebyshev sets, Klee sets, and Chebyshev centers with respect to Bregman distances: recent results and open problems*, Fixed-Point Algorithms for Inverse Problems in Science and Engineering, Springer, Optim. Appl., 49, Springer, New York, 2011, 1-21.

[5] H.H. Bauschke, X. Wang, J. Ye, X. Yuan, *Bregman distances and Chebyshev sets*, J. Approx. Theory, **159**(2009), no. 1, 3-25.

[6] J.M. Borwein, Q.J. Zhu, *Multifunctional and functional analytic techniques in nonsmooth analysis*, Nonlinear Analysis, Differential Equations and Control, NATO Sci. Ser. C Math. Phys. Sci., 528, Kluwer Acad. Publ., Dordrecht, 1999, 61-157.

[7] L.M. Bregman, *The relaxation method of finding the common point of convex sets and its application to the solution of problems in convex programming*, USSR Comput. Math. Math. Phys., **7**(1967), no. 3, 200-217.

[8] R.E. Bruck, T. Kuczumow, S. Reich, *Convergence of iterates of asymptotically nonexpansive mappings in Banach spaces with the uniform Opial property*, Colloq. Math., **65**(1993), no. 2, 169-179.

[9] D. Butnariu, Y. Censor, S. Reich, *Iterative averaging of entropic projections for solving stochastic convex feasibility problems*, Comput. Optim. Appl., **8**(1997), no. 1, 21-39.
[10] D. Butnariu, A.N. Iusem, *Totally Convex Functions for Fixed Points Computation and Infinite Dimensional Optimization*, Kluwer Academic Publ., Dordrecht, 2000.
[11] D. Butnariu, A.N. Iusem, C. Zălinescu, *On uniform convexity, total convexity and convergence of the proximal point and outer Bregman projection algorithms in Banach spaces*, J. Convex Anal., **10**(2003), no. 1, 35-61.
[12] D. Butnariu, E. Resmerita, *Bregman distances, totally convex functions, and a method for solving operator equations in Banach spaces*, Abstr. Appl. Anal., (2006), 1-39.
[13] D. Butnariu, S. Reich, A.J. Zaslavski, *There are many totally convex functions*, J. Convex Anal., **13**(2006), 623-632.
[14] Y. Censor, A. Lent, *An iterative row-action method for interval convex programming*, J. Optim. Theory Appl., **34**(1981), no. 3, 321-353.
[15] K. Goebel, W.A. Kirk, *A fixed point theorem for asymptotically nonexpansive mappings*, Proc. Amer. Math. Soc., **35**(1972), 171-174.
[16] K. Goebel, S. Reich, *Uniform Convexity, Hyperbolic Geometry, and Nonexpansive Mappings*, M. Dekker, New York, 1984.
[17] Y. Hao, *Some results on a modified Mann iterative scheme in a reflexive Banach space*, Fixed Point Theory Appl., (2013), 2013:227, 14 pp.
[18] Y. Hecai, L. Aichao, *Projection algorithms for treating asymptotically quasi-ϕ-nonexpansive mappings in the intermediate sense*, J. Inequal. Appl., (2013), 2013:265, 15 pp.
[19] T. Ibaraki, W. Takahashi, *A new projection and convergence theorems for the projections in Banach spaces*, J. Approx. Theory, **149**(2007), no. 1, 1-14.
[20] I. Inchan, *Strong convergence theorems of modified Mann iteration methods for asymptotically nonexpansive mappings in Hilbert spaces*, Int. J. Math. Anal., **2**(2008), no. 23, 1135-1145.
[21] F. Kohsaka, W. Takahashi, *Generalized nonexpansive retractions and a proximal-type algorithm in Banach spaces*, J. Nonlinear Convex Anal., **8**(2007), no. 2, 197-209.
[22] V. Martín-Márquez, S. Reich, S. Sabach, *Iterative methods for approximating fixed points of Bregman nonexpansive operators*, Discrete Contin. Dyn. Syst. Ser. S, **6**(2013), no. 4, 1043-1063.
[23] V. Martín-Márquez, S. Reich, S. Sabach, *Right Bregman nonexpansive operators in Banach spaces*, Nonlinear Anal., **75**(2012), no. 14, 5448-5465.
[24] U. Mosco, *Convergence of convex sets and of solutions of variational inequalities*, Adv. Math., **3**(1969), 510-585.
[25] R.R. Phelps, *Convex Functions, Monotone Operators, and Differentiability*, Second ed., Lecture Notes in Math., vol. 1364, Springer-Verlag, Berlin, 1993.
[26] Y. Qing, *Some results on asymptotically quasi-ϕ-nonexpansive mappings in the intermediate sense*, J. Fixed Point Theory, (2012), 2012:1, 15pp.
[27] S. Reich, *Asymptotic behavior of contractions in Banach spaces*, J. Math. Anal. Appl., **44**(1973), 57-70.
[28] S. Reich, *A weak convergence theorem for the alternating method with Bregman distances*, Theory and Applications of Nonlinear Operators of Accretive and Monotone Type, Lecture Notes Pure Appl. Math., 178, Dekker, New York, 1996, 313-318.
[29] S. Reich, S. Sabach, *Two strong convergence theorems for a proximal method in reflexive Banach spaces*, Numer. Funct. Anal. Optim., **31**(2010), no. 1, 22-44.
[30] S. Reich, S. Sabach, *Two strong convergence theorems for Bregman strongly nonexpansive operators in reflexive Banach spaces*, Nonlinear Anal., **73**(2010), no. 1, 122-135.
[31] S. Reich, S. Sabach, *Existence and approximation of fixed points of Bregman firmly nonexpansive mappings in reflexive Banach spaces*, Fixed-Point Algorithms for Inverse Problems in Science and Engineering, Springer, Optim. Appl., **49**, Springer, New York, 2011, 301-316.
[32] S. Reich, S. Sabach, *A projection method for solving nonlinear problems in reflexive Banach spaces*, J. Fixed Point Theory Appl., **9**(2011), 101-116.
[33] E. Resmerita, *On total convexity, Bregman projections and stability in Banach spaces*, J. Convex Anal., **11**(2004), no. 1, 1-16.

[34] J. Schu, *Weak and strong convergence to fixed points of asymptotically nonexpansive mappings*, Bull. Austral. Math. Soc., **43**(1991), no. 1, 153-159.
[35] S. Suantai, Y.J. Cho, P. Cholamjiak, *Halpern's Iteration for Bregman strongly nonexpansive mappings in reflexive Banach spaces*, Comput. Math. Appl., **64**(2012), 489-499.
[36] W. Takahashi, *Nonlinear Functional Analysis*, Yokohama Publishers, Yokohama, 2000.
[37] W. Takahashi, Y. Takeuchi, R. Kubota, *Strong convergence theorems by hybrid methods for families of nonexpansive mappings in Hilbert spaces*, J. Math. Anal. Appl., **341**(2008), no. 1, 276-286.
[38] W. Takahashi, J.-C. Yao, *Weak and strong convergence theorems for positively homogeneous nonexpansive mappings in Banach spaces*, Taiwanese J. Math., **15**(2011), no. 3, 961-980.

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