

APPROXIMATION OF COMMON SOLUTIONS TO PROXIMAL SPLIT FEASIBILITY PROBLEMS AND FIXED POINT PROBLEMS

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Abstract. This paper is concerned with an algorithmic solution to the proximal split feasibility problem which is also a fixed point of a k -strictly pseudocontractive mapping in Hilbert spaces. Under some assumptions on the parameters in our iterative algorithm, we first establish a strong convergence theorem for a problem of finding a point which minimizes a proper convex lower-semicontinuous function f which is also a fixed point of a k -strictly pseudocontractive mapping such that its image under a bounded linear operator A minimizes another proper convex lower-semicontinuous function g and secondly prove another strong convergence result for a problem of finding a point which minimizes a proper convex lower-semicontinuous function f which is also a fixed point of a k -strictly pseudocontractive mapping such that its image under a bounded linear operator A minimizes locally lower semicontinuous, prox-bounded and prox-regular function g . In all our results in this work, our iterative schemes are proposed with a way of selecting the step-sizes such that their implementation does not need any prior information about the operator norm because the calculation or at least an estimate of the operator norm $\|A\|$ is very difficult, if it is not an impossible task. Our result complement many recent and important results in this direction.

Key Words and Phrases: proximal split feasibility problems, Moreau-Yosida approximate, prox-regularity, k -strictly pseudocontractive mapping, fixed point, strong convergence, Hilbert spaces.

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