

OPTIMAL GRONWALL LEMMAS

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Abstract. In this paper we give some results in connection with Problem 5 and Problem 9 in [22, p. 223 and p. 226]. Here we consider some optimal Gronwall lemmas, consequences of Abstract Gronwall Lemma [20] and some Gronwall lemmas which are not optimal Gronwall lemmas (see [22, Example 6.2, p. 224] or [5, p. 226]).

Key Words and Phrases: Gronwall lemmas, optimal Gronwall lemmas, abstract Gronwall lemma, Picard operators, Volterra integral inequalities.

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