

EXISTENCE RESULTS FOR A SYSTEM OF NONLINEAR INTEGRAL EQUATIONS IN BANACH ALGEBRAS UNDER WEAK TOPOLOGY

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Abstract. This paper is devoted to the study of a coupled system of nonlinear functional integral equations in suitable Banach algebras. This system is reduced to a fixed point problem for a 2×2 block operator matrix with nonlinear inputs. Hence, certain assumptions on its entries are given under a weak topology setting. These assumptions involve in particular the De Blasi measure of weak noncompactness in order to ensure the existence of solutions.

Key Words and Phrases: integral equation, Banach algebra, weakly sequentially continuous, measure of weak noncompactness, fixed point theory.

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