COMMON SOLUTIONS TO SOME SYSTEMS OF VARIATIONAL INEQUALITIES AND FIXED POINT PROBLEMS

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Abstract. In this paper, we introduce an iterative scheme for approximating a common solution to a new system of unrelated split mixed vector variational inequality problems, multivalued variational inequality problems, and common fixed point problem for a family of nonexpansive mappings in a real Hilbert space. We prove a strong convergence theorem for the sequence generated by the proposed iterative scheme. The results presented in this paper generalize and unify previously known results in this area.

Key Words and Phrases: system of unrelated split mixed vector variational inequality problems, system of unrelated multivalued variational inequality problems, common fixed-point problem, iterative scheme.

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