TOPOLOGICAL DEGREE AND ATYPICAL BIFURCATION RESULTS FOR A CLASS OF MULTIVALUED PERTURBATIONS OF FREDHOLM MAPS IN BANACH SPACES

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Abstract. We obtain a global (atypical) bifurcation result for a semilinear inclusion problem in Banach spaces. The approach is topological, making use of a topological degree introduced by the second author for locally compact multivalued perturbations of nonlinerar Fredholm maps. We also study some specific aspects of the construction of this concept of degree.
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