APPROXIMATELY p-WRIGHT AFFINE FUNCTIONS, INNER PRODUCT SPACES AND DERIVATIONS

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Abstract. We prove a result on hyperstability (in normed spaces) of the equation that defines the p-Wright affine functions and show that it yields a simple characterization of complex inner product spaces. We also obtain in this way some inequalities describing derivations, Lie derivations and Lie homomorphisms.
Key Words and Phrases: Hyperstability, p-Wright affine function, inner product space, derivation, Lie derivation, Lie homomorphism, fixed point theorem.
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