

COMMON FIXED POINT THEOREMS OF MEIR-KEELER TYPE IN METRIC SPACES

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Abstract. This paper is a continuation of the recent work [M. Abtahi, *Fixed point theorems for Meir-Keeler type contractions in metric spaces*, *Fixed Point Theory*, 17(2016), No. 2, 225-236]. After establishing a criterion for sequences in metric spaces to be Cauchy, unified simple proofs for several known results on the existence of a common fixed point for compatible pairs of mappings of complete metric spaces satisfying a contractive condition of Meir-Keeler type are obtained. A very general common fixed point theorem, corresponding to the fixed point theorem of Proinov [P.D. Proinov, *Fixed point theorems in metric spaces*, *Nonlinear Anal.* **64** (2006) 546-557], is presented. Examples are given to support the results.

Key Words and Phrases: Common fixed points, compatible mappings, Meir-Keeler contractions, complete metric spaces.

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