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# PLANE ORBITS FOR SYNGE'S ELECTROMAGNETIC TWO-BODY PROBLEM - SMALL PERTURBED CIRCLE MOTIONS (II)

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**Abstract.** An existence of stable orbits of Synge's electromagnetic two-body problem is proved which bases on the known qualitative results. A new proof of the existence theorem for perturbed system is presented. The results obtained are applied to the system of equations of motion for two charged particles. It is shown that the parameters of the first Bohr's orbit satisfy the conditions obtained here.
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## 1. Introduction

In the previous paper [1] we have shown an existence of circle orbits of Synge's electromagnetic two-body problem [2]. On the other hand it is usually accepted [3] that N. Bohr's stationary states postulate does violate the classical electrodynamics. Moreover Synge himself shows in [2] that electromagnetic two-body problem is unstable. We have already proved in [4] that the original heuristic method of successive approximations proposed by Synge [2] cannot be substantiated. Moreover we have succeeded to split the equations of motion into two incompatible groups. It turns out that the two-dimensional case of the two-body problem is actually a Kepler problem. Therefore the result of [1] shows that the classical electrodynamics rather implies the existence of a stationary state than violates it. Here we show an existence not only of circle orbits but stable orbits asymptotically near to the circle orbits as well.

Let us consider the plane equations of motion for two charged particles in polar coordinates as they are derived from [2] in [1]:

$$\ddot{\rho}(t) = \rho(t)\dot{\varphi}(t) + \frac{Q_2}{c^3}\frac{[c^2 - \dot{\rho}^2(t)]\Delta(t)}{\rho^2(t)} \tag{1}$$

$$\ddot{\varphi}(t) = \frac{2\dot{\rho}(t)\dot{\varphi}(t)}{\rho(t)}\left[1 + \frac{Q_2\Delta(t)}{2c^3\rho(t)}\right] \tag{2}$$

where $\rho(t)$ and $\varphi(t)$ are unknown functions, $e_i$ $(i = 1, 2)$ are the charges of the moving particles, $m_i$ $(i = 1, 2)$ their rest masses, $Q_i = \dfrac{e_1 e_2}{m_i}$ $(i = 1, 2)$, $c$ - the speed of light, $\Delta(t) = \sqrt{c^2 - \dot{\rho}^2(t) - \rho^2(t)\dot{\varphi}^2(t)}$. The origin of the inertial system is associated with the one of the particles, so that $\rho(t)$ is the distance between the particles at instant $t$, while $\varphi(t)$ is the polar angle with respect to some polar axis at the same instant.

In [1] we have shown that the initial value problem for (1)-(2) has a solution which is a circle orbit, i.e. $\rho(t) = \rho_0 = \text{const}$ and $\varphi(t) = \dot{\varphi}_0 \cdot t$ where $\varphi_0 = \varphi(0) = 0$ and $\dot{\rho}_0 = \dot{\rho}(0) = 0$. In the same paper [1] one has found also conditions such that small perturbations in the initial velocities implies an escaping of the particles. The three-dimensional case is considered in [5]. In the present paper we show that under some relations between the initial conditions there exists a stable solution of (1)-(2) asymptotically tending to a circle orbit. Finally we note that [6] contains equations of motion which have solutions corresponding to the following cases: either collision or escaping of the charged particles. However, it is proved, in [7] that the system from [6] is not equivalent to the original J.L. Synge's system from [2].

In section 2 we consider a linearization of the right hand side of (1)-(2) using classical Taylor's formula at the point $(\rho_0, \dot{\rho}_0, \varphi_0, \dot{\varphi}_0)$. Then in section 3 we find solutions of the obtained linear system. In section 4 we prove an existence theorem for a nonlinear system using the fixed point method in [8] and show a continuous dependence of the solutions with respect to perturbations. Finally we give relations between initial conditions implying a stability of the hydrogen atom. The final conclusion is that Bohr's stationary states postulate is implied by classical electrodynamics.

## 2. Linearization of the system of equations of motion

Introduce the following denominations for the right hand side of (1) and (2), taking into account that they do not depend on $\varphi$:

$$P(\rho,\dot\rho,\dot\varphi) := \rho\dot\varphi^2 + \frac{Q_2}{c^3}\frac{(c^2-\dot\rho^2)\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}{\rho^2} \tag{3}$$

$$\Phi(\rho,\dot\rho,\dot\varphi) := \frac{2\dot\rho\dot\varphi}{\rho}\left[1+\frac{Q_2}{2c^3}\frac{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}{\rho}\right]. \tag{4}$$

The classical Taylor's formula applied to the right-hand sides of (3) and (4) at a neighborhood of the point $(\rho_0,\dot\rho_0,\dot\varphi_0)$ gives:

$$P(\rho,\dot\rho,\dot\varphi) = P(\rho_0,\dot\rho_0,\dot\varphi_0) + \frac{\partial P(\rho_0,\dot\varphi_0,\dot\varphi_0)}{\partial\rho}(\rho-\rho_0) + \frac{\partial P(\rho_0,\dot\rho_0,\dot\varphi_0)}{\partial\dot\rho}(\dot\rho-\dot\rho_0)$$
$$+\frac{\partial P(\rho_0,\dot\rho_0,\dot\varphi_0)}{\partial\dot\varphi}(\dot\varphi-\dot\varphi_0) + o_1(\sqrt{(\rho-\rho_0)^2+(\dot\rho-\dot\rho_0)^2+(\dot\varphi-\dot\varphi_0)^2}),$$

and

$$\Phi(\rho,\dot\rho,\dot\varphi) = \Phi(\rho_0,\dot\rho_0,\dot\varphi_0) + \frac{\partial\Phi(\rho_0,\dot\rho_0,\dot\varphi_0)}{\partial\rho}(\rho-\rho_0) + \frac{\partial\Phi(\rho_0,\dot\rho_0,\dot\varphi_0)}{\partial\dot\rho}(\dot\rho-\dot\rho_0)$$
$$+\frac{\partial\Phi(\rho_0,\dot\rho_0,\dot\varphi_0)}{\partial\dot\varphi}(\dot\varphi-\dot\varphi_0) + o_2(\sqrt{(\rho-\rho_0)^2+(\dot\rho-\dot\rho_0)^2+(\dot\varphi-\dot\varphi_0)^2}).$$

We have

$$\frac{\partial P}{\partial\rho} = \dot\varphi^2 + \frac{Q_2(c^2-\dot\rho^2)}{c^3\rho^3}\frac{\rho^2\dot\varphi^2+2\dot\rho^2-2c^2}{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}, \quad \frac{\partial P}{\partial\dot\rho} = \frac{Q_2\dot\rho}{c^3\rho^2}\frac{-3c^2+3\dot\rho^2+2\rho\dot\varphi^2}{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}},$$

$$\frac{\partial P}{\partial\dot\varphi} = 2\rho\dot\varphi - \frac{Q_2(c^2-\dot\rho^2)\dot\varphi}{c^2\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}, \quad \frac{\partial\Phi}{\partial\rho} = -\frac{2\dot\varphi}{\rho^2} - \frac{Q_2\dot\rho\dot\varphi}{c^3\rho^3}\frac{2c^2-2\dot\rho^2-\rho^2\dot\varphi^2}{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}},$$

$$\frac{\partial\Phi}{\partial\dot\rho} = \frac{2\dot\varphi}{\rho} + \frac{Q_2\dot\varphi}{c^3\rho^2}\frac{c^2-2\dot\rho^2-\rho^2\dot\varphi^2}{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}, \quad \frac{\partial\Phi}{\partial\dot\varphi} = \frac{2\dot\rho}{\rho} + \frac{Q_2\dot\rho}{c^3\rho^2}\frac{c^2-\dot\rho^2-2\rho^2\dot\varphi^2}{\sqrt{c^2-\dot\rho^2-\rho^2\dot\varphi^2}}.$$

Then (1)-(2) can be written in the form

$$\ddot\rho(t) = \rho_0\dot\varphi_0^2 + \frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}{\rho_0^2}$$
$$+\left[\dot\varphi_0^2+\frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)(\rho_0^2\dot\varphi_0^2-2c^2+2\dot\rho_0^2)}{\rho_0^3\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right](\rho-\rho_0)$$
$$+\frac{Q_2\dot\rho_0}{c^3}\frac{3\dot\rho_0^2+2\rho_0^2\dot\varphi_0^2-3c^2}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}(\dot\rho-\dot\rho_0)$$

$$+\left[2\rho_0\dot\varphi_0-\frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)\rho_0\dot\varphi_0}{\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right](\dot\varphi-\dot\varphi_0)$$

$$+o_1(\sqrt{(\rho-\rho_0)^2+(\dot\rho-\dot\rho_0)^2+(\dot\varphi_0-\dot\varphi_0)^2}), \tag{5}$$

$$\ddot\varphi(t)=\frac{2\dot\rho_0\dot\varphi_0}{\rho_0}\left(1+\frac{Q_2}{2c^3}\frac{\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}{\rho_0}\right)$$

$$+\left[\frac{2\dot\rho_0\dot\varphi_0}{\rho_0^2}+\frac{Q_2}{c^3}\frac{\dot\rho_0\dot\varphi_0(2c^2-2\dot\rho_0^2-\rho_0^2\dot\varphi_0^2)}{\rho_0^3\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right](\rho-\rho_0)$$

$$+\left[\frac{2\dot\varphi_0)}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\varphi_0(c^2-2\dot\rho_0^2-\rho_0^2\dot\varphi_0^2)}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right](\dot\rho-\dot\rho_0)$$

$$+\left[\frac{2\dot\rho_0}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\rho_0(c^2-\dot\rho_0^2-2\rho_0^2\dot\varphi_0)^2}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right](\dot\varphi-\dot\varphi_0)$$

$$+o_2(\sqrt{(\rho-\rho_0)^2+(\dot\rho-\dot\rho_0)^2+(\dot\varphi-\dot\varphi_0)^2}). \tag{6}$$

Introduce the denominations:

$$P^0=P(\rho_0,\dot\rho_0,\dot\varphi_0),\quad \Phi^0=\Phi(\rho_0,\dot\rho_0,\dot\varphi_0),$$

$$P^0_\rho=P_\rho(\rho_0\dot\rho_0,\dot\varphi_0),\quad P^0_{\dot\rho}=P_{\dot\rho}(\rho_0,\dot\rho_0,\dot\varphi_0),\quad P^0_{\dot\varphi}=P_{\dot\varphi}(\rho_0,\dot\rho_0,\dot\varphi_0),$$
$$\Phi^0_\rho=\Phi_\rho(\rho_0,\dot\rho_0,\dot\varphi_0),\quad \Phi^0_{\dot\rho}=\Phi_{\dot\rho}(\rho_0,\dot\rho_0,\dot\varphi_0),\quad \Phi^0_{\dot\varphi}=\Phi_{\dot\varphi}(\rho_0,\dot\rho_0,\dot\varphi_0).$$

Then in view of the existence of circle orbits (proved in [1]) we have $P^0=0$, $\Phi^0=0$. Therefore the system (5), (6) takes the form:

$$\ddot\rho(t)=P^0_\rho[\rho(t)-\rho_0]+P^0_{\dot\rho}[\dot\rho(t)-\dot\rho_0]+P^0_{\dot\varphi}[\dot\varphi(t)-\dot\varphi_0]+o_1 \tag{7}$$

$$\ddot\varphi(t)=\Phi^0_\rho[\rho(t)-\rho_0]+\Phi^0_{\dot\rho}[\dot\rho(t)-\dot\rho_0]+\Phi^0_{\dot\varphi}[\dot\varphi(t)-\dot\varphi_0]+o_2. \tag{8}$$

As usually the second order system (7), (8) can be represented as a first order one, by setting $\dot\rho=r(t)$, $\dot\varphi=\psi(t)$:

$$\dot\rho(t)=r(t)$$

$$\dot r(t)=P^0_\rho[\rho(t)-\rho_0]+P^0_{\dot\rho}[\dot\rho(t)-\dot\rho_0]+P^0_{\dot\varphi}[\dot\varphi(t)-\dot\varphi_0]+o_1$$

$$\dot\varphi(t)=\psi(t)$$

$$\dot\psi(t)=\Phi^0_\rho[\rho(t)-\rho_0]+\Phi^0_{\dot\rho}[\dot\rho(t)-\dot\rho_0]+\Phi^0_{\dot\varphi}[\dot\varphi(t)-\dot\varphi_0]+o_2.$$

Since the right hand sides of (1), (2) do not depend on $\varphi$ we can consider a system of three equations with respect to $\rho, r, \psi$:

$$\begin{aligned} &\dot{\rho}(t) = r(t) \\ &\dot{r}(t) = P^0_\rho[\rho(t) - \rho_0] + P^0_{\dot{\rho}}[\dot{\rho}(t) - \dot{\rho}_0] + P^0_{\dot{\varphi}}[\dot{\varphi}(t) - \dot{\varphi}_0] + o_1 \\ &\dot{\psi}(t) = \Phi^0_\rho[\rho(t) - \rho_0] + \Phi^0_{\dot{\rho}}[\dot{\rho}(t) - \dot{\rho}_0] + \Phi^0_{\dot{\varphi}}[\dot{\varphi}(t) - \dot{\varphi}_0] + o_2 \end{aligned} \tag{9}$$

## 3. Tools from the theory of perturbed differential equations

Let us consider the system

$$\dot{x}_k = F_k(t, x_1, x_2, \dots, x_n) \quad (k = 1, 2, \dots, n) \tag{10}$$

and its perturbed one

$$\dot{x}_k = F_k(t, x_1, x_2, \dots, x_n) + R_k(t, x_1, x_2, \dots, x_n) \quad (k = 1, 2, \dots, n). \tag{11}$$

We recall some known results from [9], [10]:

Let $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ be a solution of (10) and $y(t) = (y_1(t), y_2(t), \dots, y_n(t))$ be a solution of (11) for $t \geq t_0$ and

$$\|x\| = \sqrt{x_1^2 + \cdots + x_n^2}.$$

**Proposition 1.** *Let $F_k$ be Lipschitz continuous with constant $L$ and $\|R_k(t, u(t))\| \leq \delta$ for the solution $y(t)$ ($\delta = const > 0$). If the initial conditions satisfy the inequality $\|x^{(0)} - y^{(0)}\| \leq \eta$ then*

$$\|x(t) - y(t)\| \leq \eta e^{nL(t-t_0)} + \frac{\delta}{nL}[e^{nL(t-t_0)} - 1] \text{ for } t \geq t_0.$$

If we disregard the infinitely small quantities of second order in (7)-(8) we obtain a linear system. As Proposition 1 shows, the solutions of the linear system can be different from the solutions of the perturbed one by $e^{nL(t-t_0)}$, which implies a possible existence of a non-stable solution for nonlinear system (1), (2). So we consider the following linear perturbated system:

$$\dot{x}_s(t) = \sum_{j=1}^{n} p_{sj}(t) x_j(t) + X_s(t, x_1(t), \dots, x_n(t)) \quad (s = 1, 2, \dots, n) \tag{12}$$

where $X_k$ can possess a polynomial nonlinearities.

Recall that a linear system

$$\dot{x}_s(t) = \sum_{j=1}^{n} p_{sj}(t)x_j(t) \quad (s = 1, 2, \ldots, n) \tag{13}$$

is said to be regular if

$$\sum_{i=1}^{n} \lambda_i = -\chi \left( e^{-\int_0^t \sum\limits_{s=1}^{n} p_{ss}(\tau)d\tau} \right)$$

where $\lambda_1, \ldots, \lambda_n$ are characteristic values of the system and $\chi(\cdot)$ is the characteristic value of the function defined in [10]. A system is regular if for instance it is constant or periodic one.

Recall that

$$B_r^n(0) = \underbrace{B_r(0) \times \cdots \times B_r(0)}_{n\,times},$$

$$B_r(0) = \{(x_1, \ldots, x_n) \in \mathbb{R}^n : x_1^2 + \cdots + x_n^2 \le r^2\}, \quad r = const > 0$$

Let the following conditions (P) be fulfilled:

(P1) functions $p_{kj}(t) : [0, \infty) \to R^1 = (-\infty, \infty)$ are continuous and bounded;

(P2) $X_s(t, x_1, \ldots, x_n) : [0, \infty) \times B_r^n(0) \to R^1$ are continuous and $X_s(t, 0, \ldots, 0) = 0$, and

$$|X_s(t, x_1, \ldots, x_n) - X_s(t, \overline{x}_1, \ldots, \overline{x_n})| \le L \left( \sum_{j=1}^{n} |x_j - \overline{x}_j| \right) \max\{\|x\|^m, \|\overline{x}\|^m\}$$

where $m$ and $L$ are positive constants;

(P3) the system $\dot{x}_s(t) = \sum\limits_{j=1}^{n} p_{sj}(t)x_j(t)$ $(s = 1, 2, \ldots, n)$ is regular;

(P4) among characteristic numbers of (13) there are $k$ in number positive denoted by $\lambda_1, \ldots, \lambda_k$.

An existence of exponentially bounded solution of (14) is proved in [10].

In what follows we give a new straightforward proof of a known result by means of the fixed point method from [8].

**Proposition 2.** *Under conditions (P) the system (12) possesses a family of integral curves*

$$x_s = x_s(t, c_1, \ldots, c_k) \quad (s = 1, 2, \ldots, n) \tag{14}$$

*depending on $k$ arbitrary constants $c_1, \dots, c_k$ and satisfies the inequality*

$$\left(\sum_{s=1}^{n} x_s^2\right)^{\frac{1}{2}} \leq b\left(\sum_{j=1}^{k} c_j^2\right)^{\frac{1}{2}} e^{-(\lambda-\varepsilon)t} \tag{15}$$

*where $\lambda = \min\{\lambda_1, \dots, \lambda_k\} > 0$ and $\varepsilon > 0$ can be chosen arbitrary small ($\varepsilon < \lambda$) while $b = b(\varepsilon)$ is such that $\lim_{\varepsilon \to 0} b(\varepsilon) = \infty$. Inequality (15) is satisfied for those integral curves for which $\sum_{j=1}^{k} c_j^2 \leq c^2$ where $c > 0$ is sufficiently small.*

We recall first some known results from [9] and [10].

Let $Y(t)$ be a normal fundamental matrix of solutions of (13) such that its $j$-th column is a solution of (13) corresponding to the characteristic value $\lambda_j$ $(j = 1, 2, \dots, n)$. Then define the matrix $G(t, \tau)$ in the following way:

$$G(t,\tau) = \begin{cases} Y(t)G_1Y^{-1}(\tau), & 0 \leq \tau \leq t \\ \\ Y(t)G_2Y^{-1}(\tau), & 0 < t < \tau \end{cases} \tag{16}$$

where

$$G_1 = \begin{pmatrix} \underbrace{\begin{matrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{matrix}}_{k} & 0 \\ 0 & \underbrace{\begin{matrix} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{matrix}}_{n-k} \end{pmatrix},$$

$$G_2 = \left( \begin{array}{cc} \underbrace{\begin{array}{cccc} 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 0 \end{array}}_{k} & 0 \\ 0 & \underbrace{\begin{array}{cccc} -1 & 0 & \dots & 0 \\ 0 & -1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -1 \end{array}}_{n-k} \end{array} \right),$$

**Lemma 1.** *The matrix $G(t,\tau)$ has the properties:*

*1) For every $\delta > 0$, $\|G(t,\tau)\| \leq \begin{cases} \gamma e^{\delta(t+\tau)} e^{-\lambda(t-\tau)}, & t \geq \tau \geq 0 \\ \gamma e^{\delta(t+\tau)}, & \tau > t \geq 0 \end{cases}$ where $\gamma = \gamma(\delta) > 0$, $\lim\limits_{\delta \to +0} \gamma(\delta) = \infty$;*

*2) $\dfrac{\partial G(t,\tau)}{\partial \tau} = -G(t,\tau)P(\tau)$, $\dfrac{\partial G(t,\tau)}{\partial t} = G(t,\tau)P(t)$ for $t \neq \tau$ where $P(t) = \{p_{jk}(t)\}_{j,k=1}^{n}$;*

*3) $G(t,t-0) - G(t,t+0) = I$ where $I$ is the $n$-dimensional unit matrix.*

**Lemma 2.** *Every solution of (12) is a solution of the following system of integral equations*

$$x(t) = \int_{t_0}^{\infty} G(t,\tau)X(\tau,x(\tau))d\tau + G(t,t_0)c_1 \tag{17}$$

*where $c_1$ is $n$-dimensional constant vector and vice versa: every solution of (17) is a solution of (12).*

We have to prove the following

**Lemma 3.** *If the assumptions of Proposition 2 are satisfied then for every vector $c_1$ with $\|c_1\| \leq c_0$ for some $c_0 > 0$ the system (17) has a unique continuously differentiable solution for which $\|x(t)\| \leq b\|c_1\|e^{-(\lambda-\varepsilon)t}$ for $t \geq t_0$.*

**Proof.** Let us consider the set of all continuous vector functions $X = \{x = (x_1(\cdot), x_2(\cdot), \dots, x_n(\cdot))$ *such that* $e^{(\lambda-s)t}\|x\| < \infty\}$ with a metric $\rho_\lambda(x,\overline{x}) = \sup\{e^{(\lambda-\varepsilon)t}\|x-\overline{x}\| : t \geq t_0\}$. Define the operator $T : X \to X$ by the right hand side of (17):

$$T(x_1(t), x_2(t), \dots, x_n(t)) := \int_{t_0}^{\infty} G(t,\tau)X(\tau, x_1(\tau), \dots, x_n(\tau))d\tau + G(t,t_0)c_1.$$

We show that for every $\varepsilon > 0$ which satisfies $0 < \varepsilon < \dfrac{m}{m+2}\lambda$ there exists $\gamma_0 > 0$ and $c_0 > 0$ such that the operator $T(\cdot)$ maps the set $M(\varepsilon, \gamma_0) = \{(x_1, \dots, x_n) \in X : \|x\| \le \gamma_0 e^{-(\lambda-\varepsilon)t}, t \ge t_0\}$ into itself when $\|c_1\| \le c_0$. It is easy to check that the set $M$ is bounded with respect to the above metric.

Indeed let us choose $\gamma_0 < r$ ($r$ is from condition (P2)). Then the function $X(t, x_1(t), x_2(t), \dots, x_n(t))$ is defined for every $(x_1, \dots, x_n) \in M(\varepsilon, \gamma_0)$ and satisfies the inequality

$$\|X(t, x_1(t), x_2(t), \dots, x_n(t))\| \le Ln\gamma_0^{m+1} e^{-(m+1)(\lambda-\varepsilon)t}, \quad t \ge t_0.$$

In view of Lemma 1 we obtain

$$\begin{aligned}
&\|T(x_1(t), x_2(t), \dots, x_n(t))\| \\
&\le \left\| \int_{t_0}^{\infty} G(t,\tau) X(\tau, x_1(\tau), x_2(\tau), \dots, x_n(\tau)) d\tau \right\| + \|G(t,t_0)c_1\| \\
&\le \int_{t_0}^{\infty} \|G(t,\tau)\| \|X(\tau, x_1(\tau), x_2(\tau), \dots, x_n(\tau)\| d\tau \\
&\quad + \|G(t,t_0)\| \|c_1\| \\
&\le Ln\gamma_0^{m+1} \left( \int_{t_0}^{t} \|G(t,\tau)\| e^{-(m+1)(\lambda-\varepsilon)\tau} d\tau + \int_{t}^{\infty} \|G(t,\tau)\| e^{-(m+1)(\lambda-\varepsilon)\tau} d\tau \right) \\
&\quad + \|G(t,t_0)\| \|c_1\| \\
&\le Ln\gamma_0^{m+1}\gamma \left( \int_{t_0}^{t} e^{-\lambda(t-\tau)} e^{\delta(t+\tau)} e^{-(m+1)(\lambda-\varepsilon)\tau} d\tau + \int_{t}^{\infty} e^{\delta(t+\tau)} e^{-(m+1)(\lambda-\varepsilon)\tau} d\tau \right) \\
&\quad + \|G(t,t_0\| \|c_1\| \\
&= Ln\gamma_0^{m+1}\gamma \left( e^{-(\lambda-\delta)t} \frac{e^{[\lambda+\delta-(m+1)(\lambda-\varepsilon)]\tau}}{\lambda+\delta-(m+1)(\lambda-\varepsilon)} \Big|_{t_0}^{t} + e^{\delta t} \frac{e^{[\delta-(m+1)(\lambda-\varepsilon)]\tau}}{\delta-(m+1)(\lambda-\varepsilon)} \Big|_{t}^{\infty} \right) \\
&\quad + \|c_1\| \gamma e^{\delta(t+t_0)} e^{-\lambda(t-t_0)} \\
&\le Ln\gamma_0^{m+1}\gamma \left( \frac{e^{\lambda+\delta-(m+1)(\lambda-\varepsilon)t} - e^{[\lambda+\delta-(m+1)(\lambda-\varepsilon)]t_0}}{\lambda+\delta-(m+1)(\lambda-\varepsilon)} e^{-(\lambda-\delta)t} - e^{\delta t} \frac{e^{[\delta-(m+1)(\lambda-\varepsilon)]t}}{\delta-(m+1)(\lambda-\varepsilon)} \right) \\
&\quad + \|c_1\| \gamma e^{-(\lambda-\delta)(t-t_0)} e^{2\delta t_0}.
\end{aligned}$$

We can choose $\delta$ such that $0 < \delta < \varepsilon$ and

$$\delta - (m+1)(\lambda-\varepsilon) < \varepsilon - (m+1)\lambda + (m+1)\varepsilon < (m+2)\varepsilon - (m+1)\lambda < -\lambda < 0$$

and

$$\lambda + \delta - (m+1)(\lambda-\varepsilon) < \lambda + \varepsilon - (m+1)\lambda + (m+1)\varepsilon$$

$$= -m\lambda + (m+2)\varepsilon < -m\lambda + m\lambda = 0.$$

Consequently since $e^{-(\lambda-\delta)t} < e^{-(\lambda-\varepsilon)t}$ we obtain

$$\|T(x_1,\ldots,x_n)\|$$
$$\le Ln\gamma_0^{m+1}\gamma e^{-(\lambda-\varepsilon)t}\left(\frac{e^{[\lambda+\delta-(m+1)(\lambda-\varepsilon)]t_0}}{(m+1)(\lambda-\varepsilon)-\lambda-\delta} + \frac{1}{(m+1)(\lambda-\varepsilon)-\delta}\right)$$
$$+\|c_1\|\gamma e^{-(\lambda-\delta)t}e^{(\lambda+\delta)t_0}$$
$$\le\left[Ln\gamma_0^{m+1}\gamma\left(\frac{e^{[\lambda+\delta-(m+1)(\lambda-\varepsilon)]t_0}}{(m+1)(\lambda-\varepsilon)-\lambda-\delta} + \frac{1}{(m+1)(\lambda-\varepsilon)-\delta}\right)+ \|c_1\|\gamma e^{(\lambda+\delta)t_0}\right]e^{-(\lambda-\varepsilon)t}$$

Choosing $c_0 = \gamma_0^{m+1}$ for $\|c_1\| \le c_0$ we have

$$\|T(x_1,\ldots,x_n)\|$$
$$\le \gamma_0^{m+1}\gamma\left[nL\left(\frac{e^{(\delta+\varepsilon)r_0}}{(m+1)(\lambda-\varepsilon)-\lambda-\delta} + \frac{1}{(m+1)(\lambda-\varepsilon)-\delta}\right) + e^{(\lambda+\delta)t_0}\right]e^{-(\lambda-\varepsilon)t}$$
$$\le \gamma_0^{m+1}\gamma\left[nL\left(\frac{e^{2\varepsilon t_0}}{m\lambda-(m+2)\varepsilon} + \frac{1}{(m+1)\lambda-(m+2)\varepsilon}\right) + e^{(m+1)(\lambda-\varepsilon)t_0}\right]e^{-(\lambda-\varepsilon)t}$$
$$\le \gamma_0^{m+1}\gamma\left[nL\left(\frac{e^{2\varepsilon t_0}}{m\lambda-(m+2)\varepsilon} + \frac{1}{\lambda}\right) + e^{(m+1)(\lambda-\varepsilon)t_0}\right]e^{-(\lambda-\varepsilon)t}.$$

Taking $\gamma_0 > 0$ sufficiently small we obtain $\|T(x_1,\ldots,x_n)\| \le \gamma_0 e^{-(\lambda-\varepsilon)t}$ which implies that $T$ maps $M(\varepsilon,\gamma_0)$ into itself.

It remains to show that $T$ is contractive operator. Indeed for $0 < \delta < \varepsilon$:

$$\|T(x_1,\ldots,x_n) - T(\overline{x}_1,\ldots,\overline{x}_n)\|$$
$$\le\int_{t_0}^{\infty}\|G(t,\tau)\|\|X(\tau,x_1(\tau),x_2(\tau),\ldots,x_n(\tau)) - X(\tau,\overline{x}_1(\tau),\overline{x}_2(\tau),\ldots,\overline{x}_n(\tau))\|d\tau$$
$$\le \int_{t-0}^{\infty}\|G(t,\tau)\|L\sum_{j=1}^{n}|x_j(\tau)-\overline{x}_j(\tau)|\|x\|^m\|\overline{x}\|^m d\tau$$
$$\le L\int_{t_0}^{t}\|G(t,\tau)\||x(\tau)-\overline{x}(\tau)|\sqrt{n}\gamma_0^m e^{-m(\lambda-\varepsilon)\tau}\gamma_0^m e^{-m(\lambda-\varepsilon)\tau}d\tau$$
$$+L\int_{t}^{\infty}\|G(t,\tau)\|\|x(\tau)-\overline{x}(\tau)\|\sqrt{n}\gamma_0^{2m}e^{-2m(\lambda-\varepsilon)\tau}d\tau$$
$$\le L\sqrt{n}\gamma\gamma_0^{2m}\rho_\lambda(x,\overline{x})\int_{t_0}^{t}e^{-(\lambda-\varepsilon)\tau}e^{-2m(\lambda-\varepsilon)\tau}e^{\delta(t+\tau)}e^{-\lambda(t-\tau)}d\tau$$
$$+L\sqrt{n}\gamma\gamma_0^{2m}\rho_\lambda(x,\overline{x})\int_{t}^{\infty}e^{-(\lambda-\varepsilon)\tau}e^{-2m(\lambda-\varepsilon)\tau}e^{\delta(t+\tau)}d\tau$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}\rho_\lambda(x,\overline{x})e^{-(\lambda-\delta)t}\int_{t_0}^{t} e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+(\delta_\lambda)]\tau}d\tau$$

$$+L\sqrt{n}\gamma\gamma_0^{2m}\rho_\lambda(x,\overline{x})e^{\delta t}\int_{t}^{\infty} e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+\delta]\tau}d\tau$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}\left[e^{-(\lambda-\varepsilon)t}\frac{e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+(\delta+\lambda)]t}-e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+(\delta+\lambda)]t_0}}{-(2m+1)(\lambda-\varepsilon)+\delta+\lambda}\right.$$

$$\left.+e^{\delta t}e^{-(\lambda-\varepsilon)t}\frac{-e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+\delta]t}}{-2m(\lambda-\varepsilon)+\delta+\varepsilon-\lambda}\right]\rho_\lambda(x,\overline{x})$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}e^{-(\lambda-\varepsilon)t}\left[\frac{e^{[-(2m+1)(\lambda-\varepsilon)+\delta+\lambda]t_0}-e^{[-(\lambda-\varepsilon)-2m(\lambda-\varepsilon)+\delta+\lambda]t}}{(2m+1)(\lambda-\varepsilon)-\delta-\lambda}\right.$$

$$\left.+\frac{e^{[-(2m+1)(\lambda-\varepsilon)+2\delta]t}}{(2m+1)(\lambda-\varepsilon)-\delta}\right]\rho_\lambda(x,\overline{x}).$$

Since $0<\varepsilon<\dfrac{m}{m+2}\lambda$ the following inequalities are fulfilled

$$-(2m+1)(\lambda-\varepsilon)+\delta+\lambda<0,\quad -(2m+1)(\lambda-\varepsilon)+\delta<-\lambda<0$$

and

$$-(2m+1)(\lambda-\varepsilon)+2\delta<\delta-\lambda<\varepsilon-\lambda<0$$

we obtain

$$\|T(x_1,\dots,x_n)-T(\overline{x}_1,\dots,\overline{x}_n)\|$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}e^{-(\lambda-\varepsilon)t}\left[\frac{2}{(2m+1)(\lambda-\varepsilon)-\delta-\lambda}+\frac{1}{(2m+1)(\lambda-\varepsilon)-\delta}\right]\rho_\lambda(x,\overline{x}).$$

Multiplying the last inequality by $e^{(\lambda-\varepsilon)t}$ and taking the supremum we have

$$\rho_\lambda(Tx,T\overline{x})$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}\left[\frac{2}{(2m+1)(\lambda-\varepsilon)-\delta-\lambda}+\frac{1}{(2m+1)(\lambda-\varepsilon)-\delta}\right]\rho_\lambda(x,\overline{x})$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}\left[\frac{2}{2m\lambda-2(m+1)\varepsilon}+\frac{1}{(2m+1)\lambda-2(m+1)\varepsilon}\right]\rho_\lambda(x,\overline{x})$$

$$\le L\sqrt{n}\gamma\gamma_0^{2m}\left(\frac{1}{\varepsilon}+\frac{1}{\lambda}\right)\rho_\lambda(x,\overline{x}).$$

We can choose $\gamma_0$ so small that the expression to satisfy the inequality

$$L\sqrt{n}\gamma\gamma_0^{2m}\left[\frac{1}{\varepsilon}+\frac{1}{\lambda}\right]<1.$$

Therefore $T$ is contractive operator and in view of a particular case of Theorem 2 from [8] $T$ has a unique fixed point which is a solution of (19).

Lemma 3 is thus proved.

The proof of Proposition 2 can be completed as in [10].

In order to apply Proposition 2 to the initial value problem (1)-(2) we will calculate the eigenvalues of the linearized system of (9). Let us put

$$\rho(t) - \rho_0 = \xi(t), \quad r(t) - r_0 = \eta(t), \quad \psi(t) - \psi_0 = \chi(t)$$

where $r_0 = \dot{\rho}_0$, $\psi_0 = \dot{\varphi}_0$. Then (1)-(2) can be presented in the following way:

$$\begin{cases} \dfrac{d}{dt}(\rho(t) - \rho_0) = r(t) - r_0 \\[2ex] \dfrac{d}{dt}(r(t) - r) = P^0_\rho(\rho(t) - \rho_0) + P^0_r(r(t) - r) + P^0_\psi(\psi(t) - \psi_0) \\[2ex] \dfrac{d}{dt}(\psi(t) - \psi_0) = \Phi^0_\rho(\rho(t) - \rho_0) + \Phi^0_r(r(t) - r) + \Phi^0_\psi(\psi(t) - \psi_0) \end{cases}$$

or

$$\begin{cases} \dot{\xi}(t) = \eta(t) \\[2ex] \dot{\eta}(t) = P^0_\rho \xi(t) + P^0_r \eta(t) + P^0_\psi \chi(t) \\[2ex] \dot{\chi}(t) = \Phi^0_\rho \xi(t) + \Phi^0_r \eta(t) + \Phi^0_\psi \chi(t). \end{cases}$$

It is known that the eigenvalues are the roots of the characteristic equation

$$\begin{vmatrix} -\lambda & 1 & 0 \\[2ex] P^0_\rho & P^0_r - \lambda & P^0_\psi \\[2ex] \Phi^0_\rho & \Phi^0_r & \Phi^0_\psi - \lambda \end{vmatrix} = 0$$

or

$$F(\lambda) = \lambda^3 - (P^0_r + \Phi^0_\psi)\lambda^2 + (P^0_r \Phi^0_\psi - P^0_\psi \Phi^0_r - P^0_\psi)\lambda + P^0_\rho \Phi^0_\psi - P^0_\psi \Phi^0_\rho = 0. \quad (18)$$

It is not difficulty to find conditions for the existence of at least one positive real root $\lambda_1$ of the above equation. We can apply Proposition 2 and according to it

$$\lim_{t\to\infty} \xi(t) = 0, \quad \lim_{t\to\infty} \eta(t) = 0, \quad \lim_{t\to\infty} \chi(t) = 0 \ \Rightarrow$$

$$\lim_{t\to\infty}\rho(t)=\dot\rho_0,\quad \lim_{t\to\infty}\psi(t)=\psi_0\ \Rightarrow\ \lim_{t\to\infty}[\varphi(t)-\dot\varphi_0 t]=0.$$

So we have shown that there are stable orbits different from circle ones [1].

**Conclusion remark.** We have obtained relations between initial data which imply the existence of stable orbits of electromagnetic two-body problem or the existence of stable hydrogen atom.

We give now an explicit condition for the existence of positive root of the characteristic equation even though they can be found in every textbook in algebra.

Indeed, if for instance $F'(\lambda)>0$ and $F(0)<0$ then (18) has a unique positive root. Since

$$F'(\lambda)=3\lambda^2-2(P_r^0+\Phi_\psi^0)\lambda+(P_r^0\Phi_\psi^0-P_\psi^0\Phi_r^0-P_\rho^0)$$

then inequalities

$$(P_r^0+\Phi_\psi^0)^2-3(P_r^0\Phi_\psi^0-\Phi_\psi^0\Phi_r^0-P_\rho^0)<0 \text{ and } P_\rho^0\Phi_\psi^0-P_\psi^0\Phi_\rho^0<0$$

or

$$\left(\frac{Q_2\dot\rho_0}{c^3}\frac{3\dot\rho_0^2+2\rho_0^2\dot\varphi_0^2-3c^2}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}+\frac{2\dot\rho_0}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\rho_0(c^2-\dot\rho_0^2-2\rho_0^2\dot\varphi_0^2)}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)^2$$
$$-3\frac{Q_2\dot\rho_0}{c^3}\frac{3\dot\rho_0^2+2\rho_0^2\dot\varphi_0^2-3c^2}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\left(\frac{2\dot\rho_0)}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\rho_0(c^2-\dot\rho_0^2-2\rho_0^2\dot\varphi_0^2)}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)$$
$$+3\left[2\rho_0\dot\varphi_0-\frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)\rho_0\dot\varphi_0}{\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right]\left[\frac{2\dot\varphi_0}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\varphi_0(c^2-2\dot\rho_0^2-\rho_0^2\dot\varphi_0^2)}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right]$$
$$+3\dot\varphi_0^2+\frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)(\rho_0^2\dot\varphi_0^2-2c^2+2\dot\rho_0^2)}{\rho_0^3\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}<0$$

and

$$\left(\dot\varphi_0^2+\frac{Q_2}{c^3}\frac{(c^2-\dot\rho_0^2)(\rho_0^2\dot\varphi_0^2-2c^2+2\dot\rho_0^2)}{\rho_0^3\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)\left(\frac{2\dot\rho_0}{\rho_0}+\frac{Q_2}{c^3}\frac{\dot\rho_0(c^2-\dot\rho_0^2-2\rho_0^2\dot\varphi_0^2)}{\rho_0^2\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)$$
$$-\left(2\rho_0\dot\varphi_0-\frac{Q_2}{c^3}\frac{(c^2-_0^2)\rho_0\dot\varphi_0}{\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)\left(\frac{2\dot\rho_0\dot\varphi_0}{\rho_0^2}+\frac{Q_2}{c^3}\frac{\dot\rho_0\dot\varphi_0(2c^2-2\dot\rho_0^2-\rho_0^2\dot\varphi_0^2)}{\rho_0^3\sqrt{c^2-\dot\rho_0^2-\dot\rho_0^2\dot\varphi_0^2}}\right)<0$$

imply an existence of an unique positive root and hence a stable hydrogen atom.

We give such conditions also for the particular case $\dot{\rho}_0 = 0$. Then

$$P^0_\rho = P_\rho(\rho_0, 0, \dot{\varphi}_0) = \dot{\varphi}_0^2 + \frac{Q_2}{c\rho_0^3}\frac{\rho_0^2\dot{\varphi}_0^2 - 2c^2}{\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}},$$

$$P^0_{\dot{\rho}} = P_{\dot{\rho}}(\rho_0, 0, \dot{\varphi}_0) = 0,$$

$$P^0_{\dot{\varphi}} = P_{\dot{\varphi}} = \dot{\varphi}_0\left(2\rho_0 - \frac{Q_2}{c\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}\right)$$

and therefore

$$\Phi^0_\rho = \Phi_\rho(\rho_0, 0, \dot{\varphi}_0) = 0,$$

$$\Phi^0_{\dot{\rho}} = \Phi_{\dot{\rho}}(\rho_0, 0, \dot{\varphi}_0) = \frac{2\dot{\varphi}_0}{\rho_0} + \frac{Q_2}{c^3}\frac{\dot{\varphi}_0\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}{\rho_0^2},$$

$$\Phi^0_{\dot{\varphi}} = \Phi_{\dot{\varphi}}(\rho_0, 0, \dot{\varphi}_0) = 0$$

In this case the characteristic equation takes the form:

$$\lambda^3 - \left[\left(2\rho_0\dot{\varphi}_0 - \frac{Q_2\dot{\varphi}_0}{c\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}\right)\left(\frac{2\dot{\varphi}_0}{\rho_0} + \frac{Q_2}{c^2}\frac{\dot{\varphi}_0\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}{\rho_0^2}\right) + \dot{\varphi}_0^2\right.$$
$$\left. + \frac{Q_2}{c\rho_0^3}\frac{\rho_0^2\dot{\varphi}_0^2 - 2c^2}{\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}\right]\lambda = 0$$

We need an existence of positive roots of the above equation. But it obviously is

$$\lambda = \sqrt{\left(2\rho_0\dot{\varphi}_0 - \frac{Q_2\dot{\varphi}_0}{c\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}\right)\left(\frac{2\dot{\varphi}_0}{\rho_0} + \frac{Q_2}{c^3}\frac{\dot{\varphi}_0\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}{\rho_0^2}\right) + \dot{\varphi}_0^2 + \frac{Q_2}{c\rho_0^3}\frac{\rho_0^2\dot{\varphi}_0^2 - 2c^2}{\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}}$$

provided

$$\left(2\rho_0\dot{\varphi}_0 - \frac{Q_2\dot{\varphi}_0}{c\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}\right)\left(\frac{2\dot{\varphi}_0}{\rho_0} + \frac{Q_2}{c^3}\frac{\dot{\varphi}_0\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}}{\rho_0^2}\right)$$
$$+\dot{\varphi}_0^2 + \frac{Q_2}{c\rho_0^3}\frac{\rho_0^2\dot{\varphi}_0^2 - 2c^2}{\sqrt{c^2 - \rho_0^2\dot{\varphi}_0^2}} > 0 \qquad (19)$$

Finally, we have to show that inequality (19) is fulfilled for the first Bohr orbit $\rho_0 = 5,3\cdot 10^{-11}$m, $\rho_0\dot{\varphi}_0 = \dfrac{c}{137} = 2,2\cdot 10^6$m/s, $Q_2 = -\dfrac{e^2}{m_e} = -(1,6\cdot 10^{-19})^2/9\cdot 10^{-31} = -2,84\cdot 10^{-8}$. It is easy to verify that inequality (19) is satisfied for these values.

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