GRÜSS-TYPE INEQUALITY FOR POSITIVE LINEAR OPERATORS WITH SECOND ORDER DITZIAN-TOTIK MODULI

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In recent paper of Acu, Gonska and Rasa ([1]) it was studied how non-multiplicativ are some linear positive operators which reproduce constant functions. Let $H_n : C[a,b] \to C[a,b]$ be such operators and for $x \in [a,b]$ we consider $L(f) = H_n(f;x)$. Denote by

$$D_n(f,g;x) := H_n(fg;x) - H_n(f;x) \cdot H_n(g;x).$$

The following result was obtained in [1] (see Theorem 4 in [1]) for a given $x \in [a,b]$.

Theorem A *If $f, g \in C[a,b]$ and $x \in [a,b]$ is fixed then it holds*

$$||D(f,g;x)|| \le \frac{1}{4} \cdot \tilde{\omega}\Big(f; 2\sqrt{2H_n((e_1-x)^2;x)}\Big) \cdot \tilde{\omega}\Big(g; 2\sqrt{2H_n((e_1-x)^2;x)}\Big). \tag{1.1}$$

If we choose $H_n = B_n$-the Bernstein operator then the last estimate gives

$$|B_n(fg;x) - B_n(f;x) \cdot B_n(g;x)| \le$$

$$\le \frac{1}{4} \cdot \tilde{\omega}\Big(f; 2\sqrt{2\frac{2x(1-x)}{n}}\Big) \cdot \tilde{\omega}\Big(g; 2\sqrt{2\frac{2x(1-x)}{n}}\Big), \tag{1.2}$$

for $f, g \in C[0,1]$. Our goal is to extend the result in Theorem A for linear positive operators which reproduce linear functions. Instead of $\tilde{\omega}$- the least concave majorant of the usual modulus of continuity we measure the non-multiplicativity of H_n in terms of the second order modulus of continuity or the second order Ditizian-Totik modulus of smoothness. Our first main result states the following:

Theorem 1. *If $f, g \in C[a,b]$, $x \in [a,b]$ is fixed and $H_n : C[a,b] \to C[a,b]$ is a positive linear operator reproducing linear functions, then the following holds*

$$|D(f,g;x)| \leq \frac{3}{2} M(f) \cdot M(g),$$

$$M(f) := \sqrt{\omega_2(f^2; \sqrt{H_n((e_1-x)^2;x)}) + 2||f|| \cdot \omega_2(f; \sqrt{H_n((e_1-x)^2;x)})}, \tag{1.3}$$

and $M(g)$ is defined analogously.

REFERENCES

[1] A. Acu, H. Gonska, I. Rasa, *Grüss- and Ostrovski-type Inequalities in Approximation Theory*, manuscript (2010).