ON GRÜSS-TYPE INEQUALITIES IN APPROXIMATION THEORY

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The classical form of Grüss' inequality gives an estimate of the difference between the integral of the product and the product of the integrals of two functions in $C[a,b]$. It was first published by G. Grüss in [2]. The aim of this presentation is to discuss Grüss-type inequalities in $C(X)$, the set of continuous functions defined on a compact metric space X. We consider a functional $L(f) := H(f;x)$, where $H : C(X) \to C(X)$ is a positive linear operator and $x \in X$ is fixed. Generalizing a result of Acu et al. [1], a quantitative Grüss-type inequality is obtained in terms of the least concave majorant of the classical modulus of continuity. The interest is in the degree of non-multiplicativity of the functional L. Moreover, we apply this inequality to various known operators, in particular those of convolution- and Shepard-type.

REFERENCES

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