

MULTIPLICATIVE PERTURBATIONS OF THE LAPLACIAN AND RELATED APPROXIMATION PROBLEMS

Francesco Altomare, Sabina Milella*,

Graziana Musceo

*Dipartimento di Matematica, Università degli Studi di Bari "A.Moro",
Via E. Orabona 4, 70125 Bari-Italia.*

[smilella@dm.uniba.it]

2000 Mathematics Subject Classification. 47D06, 41A10, 41A36,
35A35.

Keywords and phrases. Second order elliptic differential operator,
positive C_0 -semigroup, positive approximation process, integral operator,
asymptotic formula, shape preserving approximation.

In this talk we present some recent results concerning the operator

$$\alpha\Delta u(x) = \sum_{i=1}^n \alpha(x) \frac{\partial^2 u}{\partial x_i^2}(x) \quad (x \in \mathbb{R}^n).$$

We prove that, if $\alpha\Delta$ is defined on the maximal domain

$$D(\alpha\Delta) = \{u \in C_0^w(\mathbb{R}^n) \cap C^2(\mathbb{R}^n) \mid \alpha\Delta u \in C_0^w(\mathbb{R}^n)\},$$

then it is closable and its closure generates a positive quasi contractive C_0 -semigroup $(T(t))_{t \geq 0}$ which fulfils the Feller property (i.e., it leaves invariant $C_0(\mathbb{R}^n)$ and it is a contractive semigroup on it) and which is associated with a suitable probability transition function on \mathbb{R}^n and hence with a Markov process on \mathbb{R}^n . Here $\alpha, w \in C_b(\mathbb{R}^n)$ are

strictly positive functions and $C_0^w(\mathbb{R}^n)$ denotes the weighted space of function $f \in C(\mathbb{R}^n)$ such that $\lim_{\|x\| \rightarrow +\infty} w(x)f(x) = 0$.

In the case of polynomial weights, we state that the semigroup $(T(t))_{t \geq 0}$ can be approximated by means of iterates of the integral operators

$$G_n(f)(x) := \frac{1}{(2\pi)^{\frac{N}{2}}} \int_{\mathbb{R}^N} f \left(\sqrt{\frac{2\alpha(x)}{n}} y + x \right) e^{-\frac{\|y\|^2}{2}} dy$$

($f \in C_0^w(\mathbb{R}^n)$, $x \in \mathbb{R}^n$, $n \geq 1$).

Such an approximation formula allow us to disclose some properties of $(T(t))_{t \geq 0}$ and of the random variables which govern the relevant Markov process.

REFERENCES

- [1] F. Altomare, S. Milella, *On a sequence of integral operators on weighted L^p spaces*, Analysis Math. 34 (2008), 237-259.
- [2] F. Altomare, S. Milella, G. Musceo, *Multiplicative perturbations of the Laplacian and related approximation problems*, preprint 2010.