ABOUT SOME PROPERTIES OF THE MAXIMUM LIKELIHOOD ESTIMATOR AND OF THE FISHER INFORMATION

Ion Mihoc*, Cristina-Ioana Fătu

Faculty of Economics, "D. Cantemir" Christian University,
Cluj-Napoca, Str. T. Mihali 56, Romania
[ion.mihoc@cantemircluj.ro,cristina.fatu@cantemircluj.ro]

2000 Mathematics Subject Classification. 62B10, 62F10, 94A17

Keywords and phrases. Statistical estimation, exponential family, Fisher's information, information matrix, maximum likelihood estimators, asymptotic properties.

Let $\mathbf{X} = (X_1, X_2, ..., X_n)$ be a sample from the population $P \in \{P_\theta : \theta \in D_\theta\}-$ a parametric family (that is, P_θ is a known probability measure when θ is known for every θ, $\theta \in D_\theta$), where $D_\theta-$ is called the parameter space, $D_\theta \subset \mathbb{R}^k$ where k is some fixed positive integer, k is dimension of D_θ.

If $f(\mathbf{X} \mid \theta)$ is the probability density function for some model of the data, which has parameter vector $\theta = (\theta_1, \theta_2, ..., \theta_k)$ then the Fisher information matrix $\mathbf{I}_n(\theta)$ of sample size n is given by the $k \times k$ symmetric matrix whose $ij-th$ element is given by the covariance between first partial derivatives of the log-likelihood, $\mathbf{I}_n(\theta)_{ij} = Cov\left[\frac{\partial \ln f(\mathbf{X}|\theta)}{\partial \theta_i}, \frac{\partial \ln f(\mathbf{X}|\theta)}{\partial \theta_j}\right].$

In this article, under certain regularity conditions, we discuss various applications of the information matrix in statistics then we have in view the maximum likelihood estimators which have useful properties, including reparametrization-invariance, consistency, and sufficiency.