L^p-APPROXIMATION OF THE C_0-SEMIGROUP ASSOCIATED WITH A GENERALIZATION OF KANTOROVICH OPERATORS ON $[0,1]^N$

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This talk deals with the approximation properties of a new class of positive linear operators $(C_n)_{n\geq 1}$ introduced and studied in [2, 3].

More precisely, let $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ be two sequences in $[0,1]$ such that $a_n < b_n$ $(n \geq 1)$. Then, for every $n \geq 1$, the operator C_n is defined by setting

$$C_n(f)(x) = \sum_{h_1,\ldots,h_N=0}^{n} \prod_{i=1}^{N} \binom{n}{h_i} x_i^{h_i}(1-x_i)^{n-h_i}$$
$$\times \left(\frac{n+1}{b_n-a_n}\right)^N \int_{\frac{h_1+a_n}{n+1}}^{\frac{h_1+b_n}{n+1}} \cdots \int_{\frac{h_N+a_n}{n+1}}^{\frac{h_N+b_n}{n+1}} f(t_1,\ldots,t_N)\,dt_1\cdots dt_N,$$

for every $f \in L^1([0,1]^N)$ $(p \geq 1)$ and $x \in [0,1]^N$.

The operators C_n represent a generalization of the multidimensional Kantorovich operators (first introduced by Zhou in [4]) and present the advantage to allow the reconstruction of any integrable function on $[0,1]^N$ by means of its mean value on a finite numbers of sub-cells of $[0,1]^N$ which do not need to be a subdivision of $[0,1]^N$.

As showed in [2], the sequence $(C_n)_{n\geq 1}$ is a positive approximation process in $C([0,1]^N)$ as well as in $L^p([0,1]^N)$.

Moreover, this new sequence is closely related to an elliptic second order differential operator of the form

$$V_l(u)(x) := \frac{1}{2}\sum_{i=1}^{N} x_i(1-x_i)\frac{\partial^2 u}{\partial x_i^2}(x) + \sum_{i=1}^{N}\left(\frac{l}{2} - x_i\right)\frac{\partial u}{\partial x_i}(x), \qquad (1)$$

where $l \in [0,2]$, $u \in \mathcal{C}^2([0,1]^N)$ and $x = (x_i)_{1\leq i\leq N} \in [0,1]^N$.

In fact, let $(T_l(t))_{t\geq 0}$ be the Feller semigroup (pre)generated by $(V_l, \mathcal{C}^2([0,1]^N))$ (such a semigroup exists thanks to [1, Theorem 4.1]).

In [3] we prove that, under suitable assumptions, the semigroup $(T_l(t))_{t\geq 0}$ can be approximate by means of suitable iterates of the operators C_n in the space $C([0,1]^N)$. Further, in the special case of $l = 1$, it can be extended to a C_0-semigroup $(\widetilde{T}(t))_{t\geq 0}$ in the space $L^p([0,1]^N)$ for which an approximation formula in terms of iterates of C_n's holds too.

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