DIRECT SIMULATION OF THE INFINITESIMAL DYNAMICS OF SEMI-DISCRETE APPROXIMATIONS FOR CONVECTION-DIFFUSION-REACTION PROBLEMS

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We introduce a scheme for approximating solutions of convection-diffusion-reaction equations by Markov jump processes. The method of lines reduces evolution PDEs to semi-discrete approximations by systems of ODEs. Our approach is to use for this resulting system a stochastic scheme which is essentially a direct simulation of the corresponding infinitesimal dynamics. The main feature of the method is that the increment by which the components of the solution are changed is fixed, while the time steps are variable. For one single equation this principle leads to a deterministic scheme with time steps inverse proportional to the right hand side of the equation. For general multidimensional systems it can be readily seen that the same approach works only in a stochastic framework. The components are changed sequentially by the given increment and in each step we need to sample the candidate under a certain distribution. We present several results in one space dimension including free boundary problems, but the general method is simple, flexible and on uniform grids it can be formulated for general evolution partial differential equations in arbitrary space dimensions.