SIMULTANEOUS APPROXIMATION BY BERNSTEIN OPERATORS IN HÖLDER-LIPSCHITZ NORMS

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In the first part of the talk we survey some results on simultaneous approximation by the classical Bernstein operators B_n.

In the second part we focus on inequalities for simultaneous approximation involving Hölder-Lipschitz norms. There we present a quantitative result on simultaneous approximation by B_n, considered as mapping the space $C^{m,\alpha}[0,1]$ into $C^{r,\beta}[0,1]$. Here $C^{m,\alpha}[0,1]$ denotes all m-times continuously differentiable functions on $[0,1]$ whose m-th derivative satisfies a Hölder-Lipschitz condition with exponent α. This space is equipped with norm

$$||f||_{m,\alpha} := \sum_{k=0}^{m} ||f^{(k)}||_\infty + \Theta_\alpha(f^{(m)}),$$

where

$$\Theta_\alpha(f,\delta) := \sup_{x,y\in[0,1],0<|x-y|\le\delta} \frac{|f(x)-f(y)|}{|x-y|^\alpha},$$

and

$$\Theta_\alpha(f) := \sup_{0<\delta\le 1} \Theta_\alpha(f,\delta).$$

$C^{r,\beta}[0,1]$ is defined analogously. One of the main results is the following

Theorem. Let $r, m \in N_0, 0 \leq \alpha, \beta \leq 1, r \leq m$, $r + \beta \leq m + \alpha$. Then for $f \in C^{m,\alpha}$ and $n > m + 1$ one has

$$||B_n f - f||_{r,\beta} \leq c_r \cdot (n - r - 1)^{\max\left\{-1, \frac{r+\beta-m-\alpha}{2}\right\}} \cdot ||f||_{m,\alpha}.$$

Here c_r is a constant depending only on r, α, β.

We also discuss related results in the subspace $\tilde{C}^{m,\alpha}[0,1]$, consisting of all functions f for which $\Theta_\alpha(f, \delta) = o(1), \delta \to 0$.

The talk is based on joint work with J. Prestin (Lübeck), G. Tachev (Sofia) and Ding-Xuan Zhou (Hong Kong).

REFERENCES

[1] J. Bustamante, M. Jiménez, *Chebyshev and Hölder approximation*, Aportaciones Mat. Comun. **27** (2000), 23-31.

[2] D. Cárdenas-Morales, M. Jiménez-Pozo, F. Muñoz-Delgado, *Some remarks on Hölder approximation by Bernstein polynomials*, Appl. Math. Letters **19** (2006), 1118-1121.

[3] H. Gonska, J. Prestin, G. Tachev, D.-X. Zhou, *Simultaneous approximation by Bernstein operators in Hölder norms*, submitted for publication.