ON SOME QUADRATURE FORMULAS ON THE REAL LINE WITH THE HIGHEST DEGREE OF ACCURANCY AND ITS APPLICATIONS

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In this talk we consider quadrature formulas of the form

$$\int_a^b f(x)w(x)dx = \sum_{k=0}^{n} A_k f(x_k) + R_n(f) \tag{1}$$

where x_k, $k = 0, 1, \ldots, n$ are distinct points on $[a, b]$.

The quadrature formula (1) has the degree of exactness m, $m \in \mathbb{N}$, if and only if $R_n(e_i) = 0$, $i = 0, 1, \ldots, m$ and $R_n(e_{m+1}) \neq 0$.

It is well known that, for n fixed $m \leq 2n + 1$.

We study the quadrature formulas (1) for which the degree of exactness m verifies the inequality

$$m \geq 2n - 2.$$

We study the quasi-orthogonality of orthogonal polynomials and we give some results on the location on their zeros. We are interested on the case when the quadrature formulas (1) have the positive weights ($A_k > 0$, $k = 0, 1, \ldots, n$). Using quadrature formulas with one or two prescribed nodes we obtain some interesting inequalities.