ON A NEW SEQUENCE OF POSITIVE LINEAR OPERATORS ON UNBOUNDED INTERVALS

Francesco Altomare

Dipartimento di Matematica, Università degli Studi di Bari "A. Moro"
Campus Universitario, Via E. Orabona, 4
70125 Bari-Italy.
[altomare@dm.uniba.it]

Mirella Cappelletti Montano

Dipartimento di Matematica, Università degli Studi di Bari "A. Moro"
Campus Universitario, Via E. Orabona, 4
70125 Bari-Italy.
[montano@dm.uniba.it]

Vita Leonessa

Dipartimento di Matematica e Informatica, Università degli Studi della Basilicata
Viale dell'Ateneo Lucano no 10, Macchia Romana
85100 Potenza-Italy.
[vita.leonessa@unibas.it]

2000 Mathematics Subject Classification. 41A10, 41A25, 41A36

Keywords and phrases. Szász-Mirakjan-Kantorovich operator. Positive approximation process. Rate of convergence.

As it is well known, the so-called Szász-Mirakjan-Kantorovich operators (introduced by P.L. Butzer in [3]) are defined by setting, for every $n \geq 1$, $f \in L^1([0,+\infty[)$ and $x \geq 0$,

$$K_n(f)(x) := \sum^{\infty} e^{-nx}\frac{(nx)^k}{k!}\left(n\int_{\frac{k}{n}}^{\frac{k+1}{n}} f(t)\,dt\right).$$

In this talk, we present a modification of Szász-Mirakjan-Kantorovich operators, recently introduced in [2].

Namely, following an idea first developed in [1], after fixing two sequences $(a_n)_{n\geq 1}$ and $(b_n)_{n\geq 1}$ of real numbers such that $0 \leq a_n < b_n \leq 1$ $(n \geq 1)$, we consider the positive linear operator defined by setting, for every $f \in L^1([0,+\infty[)$, $x \geq 0$ and $n \geq 1$,

$$C_n(f)(x) := \sum_{k=0}^{\infty} e^{-nx} \frac{(nx)^k}{k!} \left(\frac{n}{b_n - a_n} \int_{\frac{k+a_n}{n}}^{\frac{k+b_n}{n}} f(t)\, dt \right).$$

We prove that the sequence $(C_n)_{n\geq 1}$ is a positive approximation process in several continuous function spaces as well as in $L^p([0,+\infty[)$, $p \geq 1$; in particular, the C_n's allow to reconstruct a continuous or an integrable function by means of its mean values on subintervals of $[0,+\infty[$ which do not necessarily constitute a subdivision of $[0,+\infty[$.

Finally, we present some estimates of the rate of convergence with respect to different moduli of smoothness.

REFERENCES

[1] F. Altomare, and V. Leonessa, *On a sequence of positive linear operators associated with continuous selections of Borel measures*, Mediterr. J. Math. **3** (2006), 363-382.

[2] F. Altomare, M. Cappelletti Montano and V. Leonessa, *On a modification of Szász-Mirakjan-Kantorovich operators*, preprint, 2010

[3] P.L. Butzer, *On the extensions of Bernstein polynomials to the infinite interval*, Proc. Amer. Math. Soc. **5** (1954), 547-553.