Laboratory 4: Stability of the equilibrium points

1. Consider the following mosquito model

$$x_{n+1} = \left(ax_n + bx_{n-1} \cdot e^{-x_{n-1}}\right) \cdot e^{-x_n}$$

   where $a \in (0;1)$, $b \in [0;+\infty)$. This equation describes the growth of a mosquito population. Mosquitoes lay eggs, some of which hatch as soon as conditions are favorable, while others remain dormant for a year or two. In this model, it is assumed that eggs are dormant for one year at most.

   (a) Find the equilibrium points;

   (b) Study the stability of the equilibrium points;

   (c) Make numerical simulations.

2. Flour Beetles model

$$x_{n+1} = \alpha x_n + \beta x_{n-2} \cdot e^{-c_1 x_{n-2} - c_2 x_n}$$

   where $\alpha, \beta > 0$.

   (a) Find the equilibrium points;

   (b) Study the stability of the equilibrium points;

   (c) Make numerical simulations.

3. Consider a single-species, two-age-class system, with X_n being the number of young and Y_n that of adults, in the nth time interval:

$$\begin{cases} X_{n+1} &= bY_n \\ Y_{n+1} &= cX_n + sY_n - DY_n^2 \end{cases}$$

   A proportion c of the young become adult, and the rest will die before reaching adulthood. The adults have a fecundity rate b and a density dependent survival rate $sY_n - DY_n^2$.

   (a) Find the equilibrium points;

   (b) Study the stability of the equilibrium points;

   (c) Make numerical simulations.