Laboratory 1: Modelling Change with Difference Equations

1. Write the first five terms of the following sequences:

   (a) $a_{n+1} = 3a_n$, $a_0 = 1$;

   (b) $a_{n+1} = 3a_n\,(a_n + 1)$, $a_0 = 0$;

   (c) $a_{n+1} = 2a_n^2$, $a_0 = -1$;

2. Write the first ten terms of the sequence satisfying the following difference equations and draw the corresponding graph of the generated dynamical system:

   (a) $\Delta a_n = \frac{1}{3}a_n$, $a_0 = 1$;

   (b) $\Delta a_n = 0.01 \cdot (200 - a_n)$, $a_0 = 10$;

   (c) $\Delta a_n = 1.5 \cdot (100 - a_n)$, $a_0 = 200$;

3. (Decay of Digoxin in the Bloodstream) Digoxin is used in treatment of heart disease. Doctors must prescribe an amount of medicine that keeps the concetration of digoxin in the bloodstream above an *effective level* without exceeding the *safe level*. For an initial dosage of 0.5 mg in the bloodstream , table shows the amount of digoxin a_n remaining in the bloodstream of a particular pacient after n days.

n	0	1	2	3	4	5	6	7	8
a_n	0.5	0.345	0.238	0.164	0.113	0.078	0.054	0.037	0.026

   (a) Get Δa_n and plot Δa_n versus a_n

   (b) Find a proportional constant for Δa_n and a_n.

4. The following data were obtained for the growth of a sheep population introduced into new enviroment on the island of Tasmania:

Year	1814	1824	1834	1844	1854	1864
Population	125	275	830	1200	1750	1650

   (a) Plot the data. Is there a trend?

   (b) Formulate discrete dynamical system that resonable approximate the change observed. Compare real data with estimated data obtained from the model.