

Research Grant CNCSIS PN-II-ID-525/2007

THE STUDY OF SOME VISCOUS FLUID FLOWS IN POROUS MEDIA WITH APPLICATIONS IN BIOLOGY AND MEDICINE

2007.10.01–2010.09.30

Director: Professor Mirela Kohr

Faculty of Mathematics and Computer Science

Babeş-Bolyai University, Cluj-Napoca

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Host institution of the project: Babeş-Bolyai University

A. Research team:

1. Professor Mirela Kohr (director)
2. Professor Gabriela Kohr
3. Lecturer PhD Teodor Groşan
4. Lecturer PhD Dalia Cîmpean
5. Assistant PhD Cornelia Revnic (Bercea) (member as PhD student in the period 01.10.2007–28.02.2009)
6. Elena Maria Ului (PhD student; member in the period 1.03.2009–30.09.2010)

B. Project summary:

The thematic of this project is to treat up-to-date and modern problems in the dynamics of viscous fluid flows and the theory of heat transfer in porous media, granular media and micropolar fluids, based on new results that have been obtained by experienced members of this research team, as well as by valuable mathematicians and mechanics from our country and abroad. The approached domain is very challenging and has various applications in the study of certain problems in biology, medicine and chemistry.

Research directions:

- The theoretical and computational study of some boundary value problems that describe fluid motions in isotropic porous media by using potential theory; Existence and uniqueness results in Hölder or Sobolev spaces and for domains with Lyapunov boundaries, or, more generally, Lipschitz boundaries; numerical computations by using boundary element methods, finite elements or finite volume methods.

- The study of certain problems that describe slow viscous flows in anisotropic porous media by using the methods of fundamental solutions and potential theory for Stokes and anisotropic Brinkman equations. This is a new direction, connected with recent researches in biology and medicine, e.g., fluid mechanics of reproduction systems, bacterial and biofilm dynamics, dynamics of suspensions, and filtration problems that appear in chemical and oil industry. The approached domain belongs to one of the highest interdisciplinary research areas.
- The study of viscous incompressible fluid flows on $2D$ simply connected domains, by using modern techniques of complex function theory and numerical analysis
- Analytical and numerical study of some fluid flow and heat transfer problems in porous media and micropolar fluids, by using modern numerical methods, such as Keller-Box, finite difference and finite element methods.

C. Objectives

2007

1. Analytical and numerical results for problems that describe viscous fluid flows in the presence of circular or spherical porous particles, by imposing various jump conditions across the porous/fluid interfaces.
2. Analytical and numerical results for flow and heat transfer problems in viscous fluids, porous media and micropolar fluids, by including the case of some variables physical parameters (porosity, permeability, effective viscosity, etc.)

Degree of achievement: 100%

2008

1. The study of viscous incompressible fluids in isotropic porous media or in the presence of isotropic porous particles, in domains with Lyapunov boundaries, by using boundary integral and singularity methods. The analysis of the associated boundary value problems in Holder spaces of functions.
2. Mathematical models and theoretical analysis of certain steday/unsteady incompressible viscous fluid flows and heat transfer problems on surfaces with a given geometry embedded in viscous fluids, micropolar and porous media, by using the approximations of boundary layer theory
3. Numerical analysis of steday/unsteady incompressible viscous fluid flows and heat transfer problems on surfaces with a given geometry embedded in viscous fluids, micropolar and porous media, by using the approximations of boundary layer theory

Degree of achievement: 100%

2009

1. The analysis of boundary value problems that describe viscous fluid flows in isotropic porous media in Lipschitz domains and Sobolev spaces, by using the Stokes and Brinkman models
2. Analytical and numerical analysis of some free and mixed convection flow problems in Newtonian viscous fluids, non-Newtonian (micropolar) fluids and porous media
3. Analytical or numerical study of some fluid flow and heat transfer problems in viscous (Newtonian) and micropolar fluids due to the dilatation or compression of a surface

Degree of achievement: 100%

2010

1. Analysis of some problems that describe viscous fluid flows in anisotropic porous media, by using approximative and numerical methods
2. Analysis of some problems that describe viscous fluid flows in anisotropic porous media, by using boundary integral and singularity methods
3. The study of some problems involving viscous incompressible fluid flows in simply connected domains, by using modern techniques of complex function theory of one variable

Degree of achievement: 100%

Table: Objectives and activities

D. Contributions of the PhD students

- Assistant PhD Cornelia Revnic (Bercea), Faculty of Pharmacy, "Iuliu Hațieganu" Medical University, Cluj-Napoca (member in the period 1.10.2007–28.02.2009)
- PhD student Elena Maria Ului, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca (member in the period 1.03.2009–30.09.2010)

Cornelia Revnic (Bercea)

- C. Revnic (Bercea) finished her PhD studies during the period of this grant. The public presentation of the PhD thesis *Contribution to the theory of convective flow of viscous fluid and porous media* was in December 2008 at Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca. Adviser Professor I. Pop, Babeş-Bolyai University. She has collaborated with the director of the grant to treat some problems related to viscous fluid flows in porous media. This PhD thesis deals with problems in a direct connection with the thematic area of the grant.

Research publications related to the objectives 2/2007, 2 and 3/2008:

- **C. Revnic, T. Groşan**, I. Pop, D.B. Ingham, *Free convection in a square cavity filled with a bidisperse porous medium*, [International J. Thermal Sciences](#), **48**(2009), 1876-1883 (ISI). Impact Factor/2008: 1.683
- **C. Revnic**, I. Pop, **T. Groşan**, D.B. Ingham, *Natural convection in an inclined square cavity with heated and cooled adjacent walls and filled with a porous medium*, [The Open Transport Phenomena Journal](#), **1**(2009), 20-29 (BDI).
- **C. Revnic (Bercea), T. Groşan**, I. Pop, *Heat transfer in axisymmetric stagnation flow on a thin cylinder*, [Studia Univ. Babeş-Bolyai, Mathematica](#), **53**, no. 2(2008), 119-132 (BDI).
MR2440765 (2009g:76048)

In the PhD thesis and in the above papers there are investigated the following main subjects, in connection with the thematic area of this grant:

- Problems related to fluid flows and heat transfer, as well as free or forced convection in viscous fluids and porous media. These problems also refer to the case of constant temperatures, and variable heat flow (in connection with the objectives 2/2007, 2 and 3/2008).
- Two dimensional steady inclined fluid flow in the presence of a constant magnetic field (in connection with the objectives 2 and 3/2008).
- Boundary layer flow problems, by assuming free, forced or mixed convection (in connection with the objectives 2 and 3/2008).

Research visits

- University of Leeds, invited by Prof. D.B. Ingham (July 2007).

The main results have been communicated to research seminars of Department of Mechanics and Astronomy, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, and to the following conference:

- International Conference: Semicentennial "Tiberiu Popoviciu", Numerical Analysis Institute, Cluj-Napoca, May 7-10, 2008:
C.Revnic, T. Groşan, I. Pop, *Effect of the Magnetic Field and Heat Generation on the Free Convection Flow in a Tall Cavity Filled with a Porous Medium*. The paper was published in the journal *Revue d'Analyse Numérique et de Théorie l'Approximation*, vol. 37 (2008), pp. 197-208.

Elena Maria Ului

- Elena Maria Ului is a PhD student at Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca.

- PhD thesis: *Contributions to the theory of viscous fluid flows at low Reynolds numbers*. Adviser Professor Mirela Kohr. She is collaborating with the director of this grant to elaborate all chapters in her PhD thesis. The problems that are investigated in her thesis are in a direct connection with the thematic area of this grant.
- In April 2009, Elena Maria Ului presented the following PhD report: *Flow problems in viscous fluids and porous media. Applications* (in connection with the objective 1/2009).
- The PhD report *Viscous incompressible flows at low Reynolds numbers. Applications*, September 22 2010 (in connection with the objectives 1 and 2/2010).

Research publications:

- **E.M. Ului**, *Boundary integral equations for the problem of 2D Brinkman flow past several voids*, [Studia Univ. Babeş-Bolyai, Mathematica](#), 56(2011), to appear (BDI).
- M. Kohr, G.P. Raja Sekhar, **E.M. Ului**, *Two-dimensional Stokes-Brinkman cell model. A boundary integral formulation*, preprint.

In the PhD thesis and in the above papers there are investigated the following main subjects, in connection with the thematic area of this grant:

- Existence and uniqueness results in Hölder or Sobolev spaces and on Lyapunov or, more general, Lipschitz domains, for boundary value/transmission problems that describe viscous flows in the presence of porous, fluid or solid particles. These problems are treated by using potential theory for Stokes and Brinkman equations (in connection with the objective 1/2009).
- Asymptotic methods and potential theory in the analysis of some problems related to viscous incompressible flows at low Reynolds numbers past porous or solid obstacles (in connection with the objectives 1/2009, 1 and 2/2010).
- The analysis of certain Dirichlet–transmission boundary value problems for Stokes and isotropic/anisotropic Brinkman equations (in connection with the objectives 1 and 2/2010).

The main results have been communicated to research seminars of Department of Mechanics and Astronomy, Faculty of Mathematics and Computer Science, Babeş-Bolyai University. E.M. Ului attended the following international conference:

- 7th International Conference on Applied Mathematics (ICAM 7), September 1-4 2010, Baia-Mare, Romania;
E.M. Ului, *Two-dimensional Stokes-Brinkman cell model. A boundary integral formulation* (poster).

E. Achievements

E1. ISI Publications

1. **M. Kohr**, G.P. Raja Sekhar, W.L. Wendland, *Rigorous estimates for the 2D Oseen-Brinkman transmission problem in terms of the Stokes-Brinkman expansion*, [Mathematical Methods in the Applied Sciences](#), DOI: 10.1002/mma.1334, 2010; Article published online: 12.07.2010, to appear (ISI). Impact Factor/2008: 0.717
2. **M. Kohr**, C. Pinteá, W.L. Wendland, *Stokes-Brinkman transmission problems on Lipschitz and C^1 domains in Riemannian manifolds*, [Communications on Pure and Applied Analysis](#), **9**(2010), 493-537 (ISI). Impact Factor/2008: 0.839
3. **M. Kohr**, C. Pinteá, W.L. Wendland, *Brinkman-type operators on Riemannian manifolds: Transmission problems in Lipschitz and C^1 domains*, [Potential Analysis](#), **32** (2010), 229-273 (ISI). Impact Factor/2008: 0.566
4. P. Curt, **G. Kohr**, **M. Kohr**, *Homeomorphic extension of strongly spirallike mappings in \mathbb{C}^n* , [Science in China Ser. A Mathematics](#), 53(2010), no.1, 87-100 (ISI). Impact factor/2008: 0.408. MR2594749
5. A. Postelnicu, **T. Groşan**, I. Pop, *Brinkman flow of a viscous fluid through a spherical porous medium embedded in another porous medium*, *Transport in Porous Media*, 81(2010), 89-103 (ISI). Impact Factor/2008: 0.772.
6. **M. Kohr**, J. Prakash, G.P. Raja Sekhar, W.L. Wendland, *Expansions at small Reynolds numbers for the flow past a porous circular cylinder*, [Applicable Analysis](#), **88**(2009), 1093-1114 (ISI). Impact Factor/2009: 0.613. MR2561477
7. **M. Kohr**, W.L. Wendland, *Boundary integral equations for a three-dimensional Brinkman flow problem*, [Math. Nachr.](#), **282**(2009), No. 9, 1305-1333 (ISI). Impact Factor/2008: 0.537. MR2561185
8. **M. Kohr**, W.L. Wendland, G.P. Raja Sekhar, *Boundary integral equations for two-dimensional low Reynolds number flow past a porous body*, [Mathematical Methods in the Applied Sciences](#), **32**(2009), No.8, 922-962 (ISI). Impact Factor/2008: 0.717. MR2519362 (2010d:76031)
9. **D. Cîmpean**, I. Pop, D.B. Ingham, J.H. Merkin, *Fully developed mixed convection flow between inclined parallel plates filled with a porous medium*, [Transp. Porous Med.](#), **77**(2009), 87-102 (ISI). Impact Factor/2008: 0.772. MR2485526 (2010c:76103)
10. **C. Revnic**, **T. Groşan**, I. Pop, D.B. Ingham, *Free convection in a square cavity filled with a bidisperse porous medium*, [International J. Thermal Sciences](#), **48**(2009), 1876-1883 (ISI). Impact Factor/2008: 1.683
11. I. Graham, H. Hamada, **G. Kohr**, **M. Kohr**, *Spirallike mappings and univalent subordination chains in \mathbb{C}^n* , [Ann. Scuola Norm. Sup. Pisa Classe di Scienze, Serie V](#), Vol. 7(2008), 717-740 (ISI). Impact factor/2008: 0.519. MR2483641 (2009m:32026)

12. **M. Kohr**, G.P. Raja Sekhar, W.L. Wendland, *Boundary integral equations for a three dimensional Stokes-Brinkman cell model*, [Mathematical Models and Methods in Applied Sciences](#), **18** (2008), No.12, 2055-2085 (ISI). Impact Factor/2008: 2.333. MR2477717 (2010b:74030)
13. S.R. Pop, **T. Groşan**, I. Pop, *Effect of variable viscosity on free convection flow in a horizontal porous channel with a partly heated or cooled wall*, [Revista de Chimie](#), **59**(2008), 1210-1212 (ISI). Impact Factor/2008: 0.389
14. **M. Kohr**, C. Pinteaa, W.L. Wendland, *Dirichlet - transmission problems for general Brinkman operators on Lipschitz and C^1 domains in Riemannian manifolds*, [Dyscrete and Continuous Dynamical Systems - B](#), to appear (ISI). Impact Factor/2009: 0.803
15. **D.S. Cîmpean**, D. Popa, *On the stability of the linear differential equation of higher order with constant coefficients*, [Applied Mathematics and Computation](#), to appear (ISI). Impact Factor /2009: 1.124.

E2. BDI Publications

1. **M. Kohr**, C. Pinteaa, W.L. Wendland, *On mapping properties of layer potential operators for Brinkman equations on Lipschitz domains in Riemannian manifolds*, [Mathematica \(Cluj\)](#), 52(75), no. 1 (2010), 31-45 (BDI).
2. C. Revnic, **T. Groşan**, I. Pop, *Heat transfer in axisymmetric stagnation flow on thin cylinders*, [Studia Univ. Babeş-Bolyai, Mathematica](#), 55(2010), 163-171 (BDI).
3. **E.M. Ului**, *Boundary integral equations for the problem of 2D Brinkman flow past several voids*, [Studia Univ. Babeş-Bolyai, Mathematica](#), 56(2011), to appear (BDI).
4. **C. Revnic**, I. Pop, **T. Groşan**, D.B. Ingham, *Natural convection in an inclined square cavity with heated and cooled adjacent walls and filled with a porous medium*, [The Open Transport Phenomena Journal](#), **1**(2009), 20-29 (BDI).
5. **M. Kohr**, **G. Kohr**, W.L. Wendland, *Boundary integral equations for viscous incompressible flows in porous media or past porous bodies*, [Proc. Applied Math. Mech. \(PAMM\)](#), **8**(2008), no. 1, 10891-10892 (BDI).
6. **C. Revnic**, **T. Groşan**, I. Pop, *Heat transfer in axisymmetric stagnation flow on a thin cylinder*, [Studia Univ. Babeş-Bolyai, Mathematica](#), **53**, no. 2(2008), 119-132 (BDI). MR2440765 (2009g:76048)
7. **T. Groşan**, S.R. Pop, I. Pop, *Radiation and variable viscosity effects in forced convection from a horizontal plate embedded in a porous medium*, [Studia Univ. Babeş-Bolyai, Mathematica](#), **53** (2008), no. 3, 13-23 (BDI).
8. **D. S. Cîmpean**, N. Lungu, I. Pop, *A problem of entropy generation in a channel filled with a porous medium*, [Creative Math. & Inf.](#), 17(2008), no. 3, 357-362 (BDI).
9. **D. Cîmpean**, *Flow of a micropolar fluid for a weak concentration of particles, near the stagnation point on a vertical surface*, [Automation Comput. Appl. Math.](#), **16**, no. 2(2007), 9-16 (BDI).

10. P. Curt, D. Fericean, **T. Grosan**, *Φ -like functions in two-dimensional free boundary problems*, [Mathematica \(Cluj\)](#), to appear (BDI).
11. **D.S. Cîmpean**, I. Pop, *Steady flow of a micropolar fluid in a sinusoidal channel*, [Automation Comput. Appl. Math.](#), to appear (BDI).

E3. Papers submitted for publication

1. **M. Kohr**, C. Pinteă, W.L. Wendland, *Potential analysis for pseudodifferential matrix operators in Lipschitz domains on Riemannian manifolds. Applications to Brinkman operators*, submitted.
2. I. Graham, H. Hamada, **G. Kohr**, **M. Kohr**, *Extreme points, support points and Loewner variation in several complex variables*, submitted.
3. **M. Kohr**, G.P. Raja Sekhar, **E.M. Ului**, *Two-dimensional Stokes-Brinkman cell model - A boundary integral formulation*, preprint.
4. J. Prakash, G.P. Raja Sekhar, **M. Kohr**, *Faxen's laws for arbitrary oscillatory Stokes flow past a porous sphere*, submitted.

E4. Finalized PhD thesis

- In December 2008, Revnic (Bercea) Cornelia had the public presentation of her PhD thesis: *Contribution to the Theory of Convective Flow of Viscous Fluid and Porous Media*, Faculty of Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca. Adviser Professor I. Pop.

E5. Findings

- [Report-2007](#)
- [Report-2008](#)
- [Report-2009](#)
- [Report-2010](#)
- [Presentation-2009](#)

E6. Research visits

- University of Toronto, Department of Mathematics, invited by Prof. I Graham (May 2008, August 2008, May 2009, May 2010; M. Kohr and G. Kohr)
- University of Stuttgart, Department of Applied Mathematics, invited by Prof. W.L. Wendland (March 2008, July and August 2009, July 2010; M. Kohr and G. Kohr)
- Free University of Berlin, Department of Mathematics, invited by Prof. H. Begehr (Septemebr 2008 - D. Cîmpean; October 2008 - M. Kohr, G. Kohr)

- University of Leeds, invited by Prof. D.B. Ingham (July 2008; C. Revnic and D. Cîmpean).

E7. Conferences (without financial support from this grant)

- Workshop on Interfaces in Multiphase Flow, Stuttgart, July 1st - July 2nd, 2010:
M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains - Applications* (**invited speaker**)
- NTHCA10 New Trends in Harmonic and Complex Analysis, June 29 - July 3, 2010, Bremen, Germany:
M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains. Applications* (**invited talk**)
- NTHCA10 New Trends in Harmonic and Complex Analysis, June 29 - July 3, 2010, Bremen, Germany:
G. Kohr, *Parametric representation and Loewner chains in several complex variables* (**invited talk**).
- Mini-courses in Mathematical Analysis 2010, Padova, June 21-25:
M. Kohr, *Boundary value problems for Brinkman operators on Lipschitz domains. Applications*
- 7th International Conference on Applied Mathematics (ICAM 7), September 1-4 2010, Baia-Mare, Romania:
E.M. Ului, *Two-dimensional Stokes-Brinkman cell model. A boundary integral formulation* (poster).
- International Conference on Microfluidics and Complex Flows ECM 09, Tunis, November 5-6, 2009:
M. Kohr, *Transmission problems for Stokes and Brinkman operators on arbitrary Lipschitz domains. Applications to porous media flow problems* (**invited speaker**)
- International Conference on Complex Analysis and Related Topics, The 12th Romanian-Finnish Seminar, August 17-21, 2009:
M. Kohr, G.P. Raja Sekhar, W.L. Wendland, *Boundary integral equations for two-dimensional low Reynolds number flow past a porous body*
- Modern Complex Analysis and Operator Theory and Applications, IV, El Escorial (Madrid), June 17-21, 2009:
G. Kohr, *Loewner chains and the generalized Loewner differential equation on the unit ball in \mathbb{C}^n* (**invited talk**)
- The 4th Symposium on Analysis and PDEs, Purdue University, West Lafayette (USA), May 26–29 2009
- 80-th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM), Gdansk (Poland), 9 - 13 February 2009:
M. Kohr, *Boundary integral equations for 2D low Reynolds number viscous incompressible flows past porous bodies*

- Analysis, PDEs and Applications on the occasion of the 70th birthday of Vladimir Maz'ya, June 30 - July 4 2008, Rome:
M. Kohr, *Boundary integral method for a Stokes flow past porous bodies*
- 79th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM 2008), Bremen (Germany), 31 March-4 April 2008:
M. Kohr, G. Kohr, W.L. Wendland, G.P. Raja Sekhar, *Boundary integral equations for viscous incompressible flows in porous media or past porous bodies*
- MACS - 7th Joint Conference on Mathematics and Computer Science, Babeş-Bolyai University, Cluj-Napoca, July 3-6, 2008:
S.R. Pop, T. Groşan, I. Pop, *Natural convection in a horizontal porous layer with a partly heated or cooled wall with temperature dependent viscosity*
- Stochastic Phenomena, Workshop and Summer School, Babeş-Bolyai University, Cluj-Napoca, May 26-31, 2008:
S.R. Pop, T. Groşan, I. Pop, *Natural convection in a horizontal channel filled with a porous medium*
- IEEE International Conference on Automation, Quality and Testing, Robotics (AQTR 2008), May 22-25, 2008 Cluj-Napoca:
O. Capatina, T. Groşan, R. Trîmbiţaş, *Wind potential determination in a known area*
- 6th International Conference on Applied Mathematics (ICAM6), Baia Mare, Romania, September 18-21, 2008:
D. Cîmpean, N. Lungu, I. Pop, *A problem of entropy generation in a channel filled with a porous medium*
- T. Angheluţa Seminar, Cluj-Napoca, Romania, September 10-12, 2008:
D. Cîmpean, *Entropy analysis for a forced convection flow in an inclined channel*
- International Conference: Semicentennial "Tiberiu Popoviciu", Numerical Analysis Institute, Cluj-Napoca, May 7-10, 2008:
C.Revnic, T. Groşan, I. Pop, *Effect of the Magnetic Field and Heat Generation on the Free Convection Flow in a Tall Cavity Filled with a Porous Medium.*

F. Other results

Books

1. D. Cîmpean, *Mathematical Models Applied in Engineering*, [Digital Data](#), Cluj-Napoca, 2009 (monograph; without financial support from this grant).

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