MATHEMATICA, 61 (84), N° 1, 2019, pp. 183–190

SHEETS OF CONJUGACY CLASSES IN SIMPLE ALGEBRAIC GROUPS

IULIAN I. SIMION

Abstract. For a connected reductive algebraic group G defined over an algebraically closed field of characteristic p the sheets of conjugacy classes have been parametrized by G. Carnovale and F. Esposito when p is good for G. We show that the method is independent of characteristic and that a similar parametrization is possible for all p.

MSC 2010. 14L10, 14L35, 14L40.

Key words. Algebraic groups, Jordan classes, sheets of conjugacy classes.

1. INTRODUCTION

We consider three perspectives on conjugacy classes in a connected reductive algebraic group G defined over an algebraically closed field of characteristic p: sheets of conjugacy classes, induced conjugacy classes and Jordan classes (or decomposition classes).

Induced unipotent conjugacy classes (see §2) were first described in [10] and play an important role in the representation theory of finite groups associated to G. These conjugacy classes are studied in more detail in [15]. Motivated by the need for a geometric description of the space $\mathfrak{g}/G$ of adjoint orbits in the Lie algebra $\mathfrak{g}$ of G, the similar notion of induced orbits is described in [2] when G is semisimple. The orbit-space $\mathfrak{g}/G$ is partitioned into sheets, i.e. into irreducible sets of orbits having the same dimension, which are shown to be in one-to-one correspondence with conjugacy classes of pairs $(\mathfrak{l}, \gamma)$ where $\mathfrak{l}$ is a Levi subalgebra of $\mathfrak{g}$ and γ a rigid nilpotent orbit in $[\mathfrak{l}, \mathfrak{l}]$.

In [3], under the assumption that p is good for G, the authors extend the notion of induced conjugacy classes to non-unipotent elements in G and show that sheets of conjugacy classes are in one-to-one correspondence with G-conjugacy classes of triples (M, t, γ) where M is the centralizer of a semisimple element in G, t is a coset of $Z(M)^\circ$ in $Z(M)$ such that $C_G(t)^\circ = M$ and γ is a rigid unipotent conjugacy class in M. The methods for obtaining this bijection extend the analogous results in [2] to G.

In this article we show that the mild restriction on the characteristic can be removed and that the methods in [3] lead to a similar parametrization of sheets of conjugacy classes for all p. We do this without using the extended

DOI: 10.24193/mathcluj.2019.2.08

version of induced conjugacy classes. In §2 the needed facts about centralizers of semisimple elements are described. In §3 Jordan classes are considered, the arguments leading to the classification of sheets are given and some of them are shortened. The main statement for the parametrization of sheets of conjugacy classes is given in §4.

Notation. G will denote a connected reductive algebraic group defined over an algebraically closed field of characteristic p. For $g \in G$ the semisimple and unipotent factors in the Jordan decomposition are denoted by g_s and g_u respectively. For an algebraic group H, H° is the identity component of H. If $X \subseteq G$, for the identity component of the centralizer $C_G(X)$ we use the (non-standard) notation $G(X)$. For $X \subseteq G$, we let $\mathcal{O}^G(X)$ be the union of conjugacy classes in G intersecting $X \subseteq G$ non-trivially. For an integer n, $G_{(n)}$ denotes the locally closed subset of G consisting of elements having centralizer of dimension $\dim G - n$.

2. PRELIMINARIES

2.1. Centralizers

For a subset $A \subseteq G$ denote the element-wise product $A \cdot G(G(A))$ by $\mathrm{Y}(A)$. Let $\mathrm{Y}^{\mathrm{reg}}(A)$ be the subset of elements in $\mathrm{Y}(A)$ for which the dimension of the G-conjugacy class is maximal, i.e. $\mathrm{Y}^{\mathrm{reg}}(A) = \mathrm{Y}(A) \cap G_{(n)}$ for $n = \max\{\dim \mathcal{O}^G(x) : x \in \mathrm{Y}(A)\}$.

Lemma 2.1. *For a subset $A \subseteq G$ such that $A \subseteq G(A)$ we have:*
(a) $Z(G(A))^\circ = G(G(A))$,
(b) $G(A) = G(\mathrm{Y}(A))$,
(c) If $x \in \mathrm{Y}(A)$ then $G(A) \subseteq G(x)$.
In particular, if there exists $a \in \mathrm{Y}(A)$ such that $G(a) = G(A)$ then $G(x) = G(A)$ for all $x \in \mathrm{Y}^{\mathrm{reg}}(A)$.

Proof. Since $A \subseteq G(A)$ we have $G(A) \supseteq G(G(A))$, hence $G(G(A)) = C_{G(A)}(G(A))^\circ = Z(G(A))^\circ$. For (b), as $A \subseteq \mathrm{Y}(A)$ clearly $G(A) \supseteq G(\mathrm{Y}(A))$. Since $A \subseteq G(A)$ we have $A \subseteq Z(G(A))$ and $\mathrm{Y}(A) \subseteq Z(G(A))$ (by (a)). Hence $G(\mathrm{Y}(A)) \subseteq G(A)$ since $G(A) \subseteq C_G(Z(G(A)))$. For (c) fix $x \in \mathrm{Y}(A)$. Then $x = ax_a$ for some $a \in A$ and $x_a \in G(G(A))$. For $y \in G(A)$, $x^y = a^y {x_a}^y = x \Leftrightarrow x_a^y = x_a$ so $C_G(x) \supseteq G(A)$. □

If A is a set of commuting semisimple elements in G then $A \subseteq G(A)$ [17, II§4.1]. On the other hand, if a is a unipotent element and the characteristic of k is bad for G, one no longer has $a \in G(a)$ in all cases. If a is regular unipotent then [16] showed that, in bad characteristic, $a \notin C_G(a)^\circ$. All unipotent conjugacy classes in simple algebraic groups for which the elements do not lie in the connected component of their centralizers are given in [9]. The following Lemma is part of [14, Proposition 15].

Lemma 2.2. $\mathrm{Y}^{\mathrm{reg}}(a)$ *is open dense in* $\mathrm{Y}(a)$ *for any semisimple element* $a \in G$.

Remark 2.3. Considering iteratively connected components of centralizers $G(\dots G(a)\dots)$ for a semisimple element a, the sequence obtained is

$$a \quad M \quad Z^\circ \quad L \quad Z^\circ \quad L \quad \dots$$

where $M = G(a)$ is called *pseudo-Levi* in [14], L is the *Levi-envelope* of M, i.e. a minimal Levi subgroup of some parabolic subgroup containing M (see [12, §3]) and $Z^\circ = G(G(a))$ is the connected component of the center of M and L. So $\mathrm{Y}(a)$ is a connected component of $Z(M)$.

2.2. Centralizers of semisimple elements

Assume G to be semisimple and let Φ be the set of roots with respect to some maximal torus $T \subseteq G$. For $A \subseteq T$ let $\Phi(A) = \{\alpha \in \Phi : \alpha(A) = 1\}$ and for $\Psi \subseteq \Phi$ let $G(\Psi) = \langle T, U_\alpha : \alpha \in \Psi\rangle$. It is well known that $G(A) = G(\Phi(A))$ for any subset $A \subseteq T$ [17, II§4.1].

The description of those $\Psi \subseteq \Phi$ for which there exists a semisimple element $s \in G$ such that $G(\Psi) = G(s)$ requires some more terminology. Recall that a subset $\Psi \subseteq \Phi$ is called *closed* if: (C1) it is stable under reflections in the roots of Ψ and (C2) it is stable under taking sums of roots (see for example [13, 13.1]). Note that closed subsets Ψ are root systems and that if Ψ is such that $G(\Psi) = G(s)$ for some $s \in T$ then Ψ is closed.

Let $\Delta = \{\alpha_1, \dots, \alpha_n\}$ be a set of simple roots with respect to T, let α_0 the highest root with respect to Δ and $\tilde{\Delta} = \Delta \cup \{-\alpha_0\}$. Let $X_*(T)$ be the cocharacter group of T, let $V = X_*(T) \otimes \mathbb{R}$ and let $C_0 = \{v \in V : \alpha_i(v) > 0, 1 \le i \le n, \alpha_0(v) < 1\}$ be the fundamental alcove.

Assume that G is simple and simply connected and that k is the algebraic closure of a prime field $\mathbb{F}_p$. Then $T \cong X_*(T) \otimes k^* \cong X_*(T) \otimes \mathbb{Q}_{p'}$ [4, §3.1] and the elements in $(X_*(T) \otimes \mathbb{Q}_{p'}) \cap \overline{C_0}$, called *$p'$-rational points*, are in one-to-one correspondence with semisimple conjugacy classes in G (as in [5]). For $J \subsetneq \tilde{\Delta}$, let F_J be the set of $v \in V$ such that $\alpha(v) = 0$ for $\alpha \in J - \{-\alpha_0\}$, $\alpha(v) > 0$ for $\alpha \in (\tilde{\Delta} - J) - \{-\alpha_0\}$, $\alpha_0(v) = 1$ if $-\alpha_0 \in J$ and $\alpha_0(v) < 1$ if $-\alpha_0 \notin J$ (as in [6]). From the proof of [6, Proposition 2.3], with the above assumptions, we have

Proposition 2.4 (Deriziotis' Criterion). *For a closed subset Ψ of Φ, there exists a semisimple element s with $G(\Psi) = G(s)$ if and only if Ψ has a set of simple roots Δ_Ψ such that*

(1) Δ_Ψ is conjugate under W to a subset of $\tilde{\Delta}$ and
(2) the intersection of $\overline{C_0}$ with the W-orbit of F_{Δ_Ψ} contains a p'-rational point.

In particular, for $s \in G$ semisimple, $G(s)$ equals $G(\Psi)$ with Ψ generated by the conjugate of some proper subset of $\tilde{\Delta}$ (see also [8, §2.15]). If the characteristic of k is good for G then the second condition in the above proposition is not needed. A proof for arbitrary algebraically closed fields k of good characteristic is given in [14, §9]. Note that, for the construction in the proof of Proposition 32 in *loc.cit.*, in certain cases, one can choose α_1 such that the coefficient of α_1 in α_0 is coprime to p and the statement holds for some Ψ also when p is bad for G.

2.3. Induced conjugacy classes

For a parabolic subgroup $P \subseteq G$ with unipotent radical U_P we have $P = LU_P$ for a Levi subgroup $L \subseteq P$. As G has finitely many unipotent conjugacy classes [11], there is a unique class $\mathrm{Ind}_L^P(\gamma)$ intersecting γU_P in an open dense subset of γU_P for any unipotent conjugacy class $\gamma \subseteq L$. These classes were studied in [10] and are said to be *induced* from L. We write $\mathrm{Ind}_L^G(g)$ for $\mathrm{Ind}_L^G(\mathcal{O}^L(g))$. A conjugacy class is called *rigid* if it is not induced from any proper subgroup L.

Induction for arbitrary conjugacy classes in a connected reductive group G was recently investigated in [3] and [7] where the arguments for adjoint orbits in the Lie algebras of G used in [2] are transferred to G.

The arguments in [3] build on the arguments for induced unipotent classes in [10]. It follows form [15, II§3.2 and §10.15] that these arguments do not depend on the characteristic of k. Moreover it is possible to adapt the approach in [15, II§3] to arbitrary elements since most of the key statements there are for general elements in G [15, II§2.8,§10.15].

3. JORDAN CLASSES

With notation as in §2, two conjugacy classes α, β in G are called *Jordan equivalent* if there are elements $x \in \alpha$ and $y \in \beta$ such that

$$x_s \in \mathrm{Y}(y_s), \quad G(x_s) = G(y_s) \quad \text{and} \quad x_u = y_u.$$

This is an equivalence relation on conjugacy classes - by definition of $\mathrm{Y}(y_s)$ and since $G(x_s) = G(y_s)$. The union of the elements in the equivalence class of α is called the *Jordan class* as in [3] (or *decomposition class* as in [2]) associated to α. We denote it by $\mathcal{J}^G(\alpha)$ or $\mathcal{J}^G(g)$ when we want to specify a representative $g \in \alpha$.

Lemma 3.1. *For all $g \in G$, $\mathcal{J}^G(g) = \mathcal{O}^G(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u)$. In particular Jordan classes are irreducible.*

Proof. First notice that since $x_u \in G(x_s)$ for any $x \in G$ [1, III.(9.3) Proposition], the unipotent part of the Jordan decomposition of any element in $\mathrm{Y}(g_s)g_u$ is g_u. Indeed, by 2.1.a, $\mathrm{Y}(g_s) = g_s Z(G(g_s))^\circ$ so it commutes with g_u.

We also notice that g_s belongs to $\mathrm{Y}^{\mathrm{reg}}(g_s)$. Indeed, if $x \in \mathrm{Y}(g_s)$ then $x = g_s x'$ where $[g_s, x'] = 1$ so $C_G(x) = C_G(g_s) \cap C_G(x')$ but $x' \in Z(G(g_s))$ (by Lemma 2.1.a) so $C_G(x) \supseteq G(g_s)$. It follows that $\dim \mathcal{O}^G(x) \leq \dim \mathcal{O}^G(g_s)$, hence $g_s \in \mathrm{Y}^{\mathrm{reg}}(g_s)$. This shows that $\mathcal{J}^G(g) \subseteq \mathcal{O}^G(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u)$.

Now, for the inclusion $\supseteq$, if $x \in \mathrm{Y}^{\mathrm{reg}}(g_s)g_u$ then $x_s \in \mathrm{Y}^{\mathrm{reg}}(g_s) \subseteq \mathrm{Y}(g_s)$ and $x_u = g_u$. By Lemma 2.1.c, with $A = \{g_s\}$, we have $G(g_s) \subseteq G(x_s)$. Hence $g_s, x_s \in \mathrm{Y}^{\mathrm{reg}}(g_s)$ and therefore $\dim G(g_s) = \dim G(x_s) \Rightarrow G(g_s) = G(x_s)$. For $\subseteq$, choose x such that $x_s \in \mathrm{Y}(g_s)$, $x_u = g_u$ and $G(x_s) = G(g_s)$. Then $\dim G(x_s) = \dim G(g_s)$ so $x_s \in \mathrm{Y}^{\mathrm{reg}}(g_s)$ by definition of this set.

The irreducibility of $\mathcal{J}^G(g)$ follows from the fact that it is the image of the irreducible variety $G \times \mathrm{Y}^{\mathrm{reg}}(g_s)g_u$ through the morphism $(x, y) \mapsto y^x$, where the irreducibility of $\mathrm{Y}^{\mathrm{reg}}(g_s)$ follows from Lemma 2.2. □

Remark 3.2. There is an action of G on the set of triples

$$\{(G(g_s), \mathrm{Y}(g_s), \mathcal{O}^{G(g_s)}(g_u)) : g \in G\}$$

given by simultaneous conjugation. If $\mathcal{T}$ is the set of orbits for this action, then, as in [3, Proposition 4.12], we have a bijection

$$\mathcal{T} \to \text{ Jordan classes in } G, \quad \mathcal{O}^G(G(g_s), \mathrm{Y}(g_s), \mathcal{O}^{G(g_s)}(g_u)) \mapsto \mathcal{O}^G(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u).$$

Lemma 3.3. *For $g \in G$ let L be the Levi envelope of $G(g_s)$ and $Z^\circ = Z(L)^\circ$. Then $\overline{\mathcal{J}^G(g)} = \mathcal{O}^G(\overline{\mathcal{O}^L(g)}Z^\circ U_P)$, where U_P is the unipotent radical of a parabolic subgroup of G with Levi factor L.*

Proof. We have by Lemma 3.1

$$\begin{aligned} \mathcal{J}^G(g) &= \mathcal{O}^G(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u) = \mathcal{O}^G(\mathcal{O}^{U_P}(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u)) \\ &\subseteq \mathcal{O}^G(\mathrm{Y}^{\mathrm{reg}}(g_s)g_uU_P) = \mathcal{O}^G(\mathcal{O}^L(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u)U_P). \end{aligned}$$

Moreover, the inclusion is dense since $\dim C_{U_P}(tg_u) = 0$ for $t \in \mathrm{Y}^{\mathrm{reg}}(g_s)$. Indeed $\dim C_{U_P}(tg_u) = \dim(C_{U_P}(g_u) \cap G(t))$ and $G(t) = G(g_s) \subseteq L$ so $C_{U_P}(g_u) \cap G(t) = 1$.

By Lemma 2.2, $\mathcal{O}^L(\mathrm{Y}^{\mathrm{reg}}(g_s)g_u)$ is dense in the irreducible set $\mathcal{O}^L(g)Z^\circ$, hence its closure is $\overline{\mathcal{O}^L(g)}Z^\circ$, the preimage under $\phi : L \to L/Z^\circ$ of $\overline{\mathcal{O}^{L/Z^\circ}(\phi(g))}$. Therefore $\overline{\mathcal{O}^L(g)}Z^\circ U_P$ is closed in $P = LU_P \cong L \times U_P$ and is a subset of $\overline{\mathcal{J}^G(g)}$. Moreover its G-orbit is dense in $\overline{\mathcal{J}^G(g)}$ and it is P-stable. The claim follows from [18, II.13, Lemma 2]. □

As in [3, Proposition 5.3] we have the following lemma.

Lemma 3.4. *Let n be the dimension of a conjugacy class in G. The maximal elements in $\{\mathcal{J}^G(g) : g \in G_{(n)}\}$ with respect to the closure order are those $\mathcal{J}^G(g)$ with g_u rigid in $G(g_s)$.*

Proof. For $g \in G_{(n)}$ we show that if g_u is an induced class in $G(g_s)$ then there exists $x \in G_{(n)}$ such that $g \in \overline{\mathcal{J}(x)}$. Let $\tilde{L}$ be a Levi subgroup of some proper parabolic subgroup in $G(g_s)$ such that $\mathcal{O}^{G(g_s)}(g_u) = \mathrm{Ind}_{\tilde{L}}^{G(g_s)}(\tilde{u})$ for some $\tilde{u} \in \tilde{L}$. Let $\tilde{Z}^\circ := Z(\tilde{L})^\circ$. Then $\tilde{L} = C_{G(g_s)}(\tilde{Z}^\circ)$ and $L = C_G(\tilde{Z}^\circ)$ is the Levi-envelope of $\tilde{L}$ in G.

By [14, Proposition 15], there is $t \in g_s\tilde{Z}^\circ$ such that $G(t) = C_G(g_s\tilde{Z}^\circ)^\circ = (G(g_s) \cap L)^\circ = \tilde{L}$. As $\tilde{u}$ is induced in $G(g_s)$ from $\tilde{L}$, by [10, 1.3 Theorem], $\dim C_{G(g_s)}(u) = \dim C_{\tilde{L}}(\tilde{u}) = \dim C_{G(t)}(\tilde{u})$. Since $\tilde{u} \in \tilde{L} = G(t)$ we have $[\tilde{u}, t] = 1$ so $C_G(t\tilde{u}) = C_G(t) \cap C_G(\tilde{u})$, hence $\dim C_G(t\tilde{u}) = \dim C_{C_G(t)}(\tilde{u}) = \dim C_{G(t)}(\tilde{u}) = n$ and therefore $\mathcal{O}^G(t\tilde{u}) \subseteq G_{(n)}$. Moreover, since $\tilde{Z}^\circ \supseteq Z(G(g_s))^\circ$ and $\Upsilon(t) = t\tilde{Z}^\circ \ni g_s$ we have $g \in \overline{\mathcal{J}^G(t\tilde{u})}$ by Lemma 3.3. □

4. SHEETS OF CONJUGACY CLASSES

A *sheet* (of conjugacy classes) in G is an irreducible component of $G_{(n)}$ for some $n \in \{0, \ldots, \operatorname{rank}(G)\}$. As Jordan classes are irreducible, if $\mathcal{J}^G(g)$ is maximal in $G_{(n)}$ with respect to the closure order, then $\mathcal{S}^G(g) := \overline{\mathcal{J}^G(g)}$ is a sheet and every sheet is the closure in $G_{(n)}$ of a maximal Jordan class. The set of all sheets in G are denoted by $\mathcal{S}^G$.

Let $\mathcal{T}^{\max}$ be the subset of $\mathcal{T}$ (Remark 3.2) given by triples (M, s, γ) where γ is rigid in M and s is a coset of $Z(M)^\circ = G(G(s))$ in $Z(M)$. Notice that $M = G(s)$ (since we are taking a subset of $\mathcal{T}$).

The following result is [3, Theorem 5.6] without the condition on the characteristic of k.

THEOREM 4.1. *The map* $\mathcal{O}^G(G(s), \Upsilon(s), \mathcal{O}^{G(s)}(u)) \mapsto \mathcal{S}^G(su)$ *induces a bijection* $\mathcal{T}^{\max} \to \mathcal{S}^G$.

Proof. From the discussion in the first paragraph of this section, it is enough to see that $\mathcal{O}^G(G(s), \Upsilon(s), \mathcal{O}^{G(s)}(u)) \mapsto \mathcal{J}^G(su)$ is a bijection onto maximal Jordan classes. From the definition of $\mathcal{T}^{\max}$ and Lemma 3.4 we see that it is a map onto maximal Jordan classes. To see that it is injective, suppose there is $g \in G$ such that $\mathcal{O}^G(G(g_s), \Upsilon(g_s), \mathcal{O}^{G(g_s)}(g_u))$ maps to $\mathcal{J}^G(su)$. By Lemma 3.1 we may assume that $g_u = u$ and $g_s \in \Upsilon^{\mathrm{reg}}(s)$. By Lemma 2.1 $G(g_s) = G(s)$ and it remains to show that $\Upsilon(s) = \Upsilon(g_s)$. But this follows from the fact that both of these sets are irreducible components of $Z(G(s))$ which intersect nontrivially. □

Centralizers of semisimple elements can be determined as explained in §2 and rigid orbits are described in [15]. A complete list of rigid orbit is, to the knowledge of the author, not available.

REMARK 4.2. It is clear from Lemma 3.1 that if $1 \neq z \in Z(G)$ and $g \in G$ then $\mathcal{J}^G(g) \neq \mathcal{J}^G(z)$ whenever $z \notin g_s^{-1}\Upsilon(g_s) = G(G(g_s))$. Moreover, for any isogeny $G \to G_{\mathrm{ad}}$ of a simple algebraic group to the adjoint group and a maximal $\mathcal{J}^G(g)$ in a sheet, all sheets $\mathcal{S}^G(zg)$ $(z \in Z(G))$ are mapped to a single sheet in G_{ad}.

REMARK 4.3. Note that if G is of type B_2, half the highest root lies in the closure $\overline{C_0}$ of the fundamental alcove. This element is p'-rational whenever $p \neq 2$, i.e. whenever p is a good prime for G. If $p = 2$ then there are two unipotent conjugacy classes of dimension 4 (see [15, p.233] or [9, p.121]) as opposed to one such class when p is good for G.

By [15, Prop.3.2.b) p.60], if α is induced from β (hence β intersects the Levi factor L of a proper parabolic subgroup of G) then $\dim \mathcal{B}_v^G = \dim \mathcal{B}_u^L$ where $v \in \alpha$, $u \in \beta$ and $\mathcal{B}_v^G$ is the variety of Borel subgroups in G containing v. If $p = 2$ then $\dim \mathcal{B}_u^L$ is either 0 or 1 and $\dim \mathcal{B}_v^G$ are given in [15, p.233]. We see that there are two classes with $\dim \mathcal{B}_v^G = 2$, which have to be rigid.

This suggests that the number of sheets in a connected and simply connected algebraic group is independent of p.

REFERENCES

[1] A. Borel, *Linear Algebraic Groups*, Graduate Texts in Mathematics, Vol. 126, Springer Verlag, New York, second edition, 1991.

[2] W. Borho, *Über Schichten halbeinfacher Lie-Algebren*, Invent. Math., **65** (1981), 283–317.

[3] G. Carnovale and F. Esposito, *On sheets of conjugacy classes in good characteristic*, Int. Math. Res. Not. IMRN, **4** (2012), 810–828.

[4] R. W. Carter, *Finite groups of Lie type – Conjugacy classes and complex characters*, John Wiley & Sons Ltd., Chichester, 1993.

[5] D. I. Deriziotis, *The Brauer complex of a Chevalley group*, J. Algebra, **70** (1981), 261–269.

[6] D. I. Deriziotis, *Centralizers of semisimple elements in a Chevalley group*, Comm. Algebra, **9** (1981), 1997–2014.

[7] W. Hoffmann, *Induced conjugacy classes, prehomogeneous varieties, and canonical parabolic subgroups*, arXiv:1206.3068 [math.GR], 2012.

[8] J. E. Humphreys, *Conjugacy classes in semisimple algebraic groups*, Mathematical Surveys and Monographs, Vol. 43, Amer. Math. Soc., Providence, RI, 1995.

[9] M. W. Liebeck and G. M. Seitz, *Unipotent and nilpotent classes in simple algebraic groups and Lie algebras*, Mathematical Surveys and Monographs, vol. 180, Amer. Math. Soc., Providence, RI, 2012.

[10] G. Lusztig and N. Spaltenstein, *Induced unipotent classes*, J. Lond. Math. Soc., **19** (1979), 41–52.

[11] G. Lusztig, *On the finiteness of the number of unipotent classes*, Invent. Math., **34** (1976), 201–213.

[12] G. Lusztig, *Intersection cohomology complexes on a reductive group*, Invent. Math., **75** (1984), 205–272.

[13] G. Malle and D. Testerman, *Linear algebraic groups and finite groups of Lie type*, Cambridge Studies in Advanced Mathematics, Vol. 133, Cambridge University Press, Cambridge, 2011.

[14] G. J. McNinch and E. Sommers, *Component groups of unipotent centralizers in good characteristic*, J. Algebra, **260** (2003), 323–337.

[15] N. Spaltenstein, *Classes unipotentes et sous-groupes de Borel*, Lecture Notes in Mathematics, Vol. 946, Springer Verlag, Berlin, 1982.

[16] T. A. Springer, *Some arithmetical results on semi-simple Lie algebras*, Publ. Math. Inst. Hautes Études Sci., **30** (1966), 115–141.

[17] T. A. Springer and R. Steinberg, *Conjugacy classes*, in *Seminar on Algebraic Groups and Related Finite Groups*, Lecture Notes in Mathematics, Vol. 131, Springer, Berlin, 1970, pp. 167–266.

[18] R. Steinberg, *Conjugacy classes in algebraic groups*, Lecture Notes in Mathematics, Vol. 366, Springer Verlag, Berlin, 1974.

Received June 24, 2018
Accepted August 3, 2018

Babeş-Bolyai University
Faculty of Mathematics
and Computer Science
1 M. Kogălniceanu St.
400084 Cluj-Napoca
E-mail: simion@math.ubbcluj.ro