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ROUGH STATISTICAL CONVERGENCE OF SEQUENCES OF FUZZY NUMBERS

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Abstract. In this paper we introduce the notion of rough statistical convergence in the fuzzy setting, which generalizes rough convergence of sequences of fuzzy numbers. We define the set of rough statistical limit points of a sequence of fuzzy numbers and prove some results associated with these notions.

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1. INTRODUCTION AND PRELIMINARIES

The concept of fuzzy set was introduced by L.A. Zadeh in 1965. The potential of the notion of fuzzy set was realized by different scientific groups and many researchers were interested in further investigation and its applications. It has been studied in various branches of science, where mathematics has applications. Authors interested in sequence spaces have also applied this notion and introduced different classes of sequences of fuzzy real numbers and studied their different properties. The concept of the convergence of a sequence of fuzzy numbers was introduced by Matloka [14], who proved some basic theorems. Later on, several mathematicians, such as Nanda [15], Savas [20], Tripathy and Debnath [22] and many others have generalized the concept.

The classical analysis is often based on fine behavior, valid for all points of some subsets, even if some distance tends to zero. Since many objects of the material universe and many objects represented by the digital computers cannot satisfy such requirements, the so-called rough analysis was developed as an approach to a rough world. The idea of rough convergence was first introduced by Phu [17], in finite dimensional normed linear spaces. In [17], Phu showed that the set $LIM^r x$ is bounded, closed and convex. Also he investigated the relationship between rough convergence and other types of convergence and the dependence of $LIM^r x$ with respect to the roughness degree r. Later on,

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Aytar [3] extended this concept and introduced the rough statistical convergence. Recently, in [8], we have introduced the rough convergence of fuzzy numbers based on α-level sets. In this paper, we introduce the rough statistical convergence in fuzzy setting and the r-statistical limit set of a sequence of fuzzy numbers. We note that our results are analogue to those of Phu's [17] and Aytar's [3]. The actual origin of most of our results and proof techniques is in those papers. We actually present those results in generalized form. This increases the interest in finding applications of these concepts.

Throughout the paper r denotes a non-negative real number. The sequence (x_n) in a metric space (X, d) is said to be r-convergent to a point $x_* \in X$, denoted as $x_n \xrightarrow{r} x_*$, if, given $\varepsilon > 0$, there exists a natural number n_0 such that $d(x_n, x_*) < r + \varepsilon$, $\forall n \geq n_0$, and the $r - limit$ set of (x_n) is defined as $LIM^r x_n = \{x_* \in X : x_n \xrightarrow{r} x_*\}$. This is the rough convergence with r as roughness degree. For $r = 0$, we get the usual convergence in a metric space. A sequence (x_n) is said to be $r-$convergent if $LIM^r x_n \neq \emptyset$.

We first recall some basic notions in the theory of fuzzy numbers. We denote by D, the set of all closed and bounded intervals on the real line $\mathbb{R}$, i.e. $D = \{A \subset R : A = [A_l, A_u]\}$. For $A, B \in D$ we have $A \leq B$ if and only if $A_l \leq B_l$, $A_u \leq B_u$ and $d = \max\{|A_l - B_l|, |A_u - B_u|\}$. Then (D, d) forms a complete metric space.

Definition 1.1. A *fuzzy number* is a function X from $\mathbb{R}$ to $[0, 1]$, which satisfies the following conditions:
(i) X is normal.
(ii) X is fuzzy convex.
(iii) X is upper semi-continuous.
(iv) The closure of the set $\{x \in \mathbb{R} : X(x) > 0\}$ is compact.

Properties (i)-(iv) imply that, for each $\alpha \in (0, 1]$, the α-level set $X^\alpha = \{x \in R : X(x) \geq \alpha\} = [X_l^\alpha, X_u^\alpha]$ is a non-empty compact convex subset of $\mathbb{R}$. The 0-level set is the class of the strong 0-cut, i.e. $cl\{x \in \mathbb{R} : X(t) > \alpha\}$. Let $L(\mathbb{R})$ denote the set of all fuzzy numbers. Define a map on $L(\mathbb{R})$, by $\overline{d}(X, Y) = \sup_{\alpha \in (0,1]} d(X^\alpha, Y^\alpha)$. Then $\left(L(\mathbb{R}), \overline{d}\right)$ forms a complete metric space (see [19]).

Definition 1.2. A subset A of N is said to *have density* $\delta(A)$ if $\delta(A) = \lim_{n\to\infty} \frac{1}{n} \cdot |A|$, where $|A|$ denotes the number of elements in A.

Definition 1.3 ([16]). A fuzzy number sequence $X = (X_n)$ is said to be *statistically convergent to the fuzzy number* X_0, if, for every $\varepsilon > 0$,

$$\delta\left(\left\{n \in N : \bar{d}(X_n, X_0) \geq \varepsilon\right\}\right) = 0,$$

and X_0 is called the *statistical limit* of X, written as $st - lim\, X_n = X_0$.

Definition 1.4 ([2]). If $\left(X_{k(j)}\right)$ is a subsequence of $X = (X_n)$ and $K = \{k(j) \in N : j \in N\}$, then we abbreviate $\left(X_{k(j)}\right)$ by $(X)_K$, which, in the case

$\delta(K) = 0$, is called a *subsequence of density zero* or a *thin subsequence*. On the other hand, $(X)_K$ is a nonthin subsequence of X, if $\delta(K) \neq 0$.

DEFINITION 1.5 ([2]). The fuzzy number ν is called *statistical limit point of sequence of fuzzy number* $X = (X_n)$, provided that there is a nonthin subsequence of X that converges to ν. Let Λ_X denote the set of statistical limit points of the sequence X.

DEFINITION 1.6 ([2]). The fuzzy number μ is called *statistical cluster point of sequence of fuzzy number* $X = (X_n)$ if $\delta\left(\left\{n \in N : \bar{d}(X_n, \mu) < \varepsilon\right\}\right) > 0$ for every $\varepsilon > 0$. Let Γ_X denote the set of statistical cluster points of X.

2. MAIN RESULT

DEFINITION 2.1. Let (X_n) be a sequence of fuzzy numbers in the metric space $\left(L(\mathbb{R}), \bar{d}\right)$ and r be a non-negative real number. (X_n) is said to be *r-statistically convergent to* X_* if, for all $\varepsilon > 0$,

$$\delta\left(\left\{n \in N : \bar{d}(X_n, X_*) \geq r + \varepsilon\right\}\right) = 0.$$

This is the r-statistical convergence with r as roughness degree. For $r = 0$ we get the statistical convergence.

EXAMPLE 2.2. Let

$$X_n(t) = \begin{cases} \bar{1}, & \text{for } n = 2^{2k+1} (k \in N) \\ \mu'_n(t), & \text{for } n = k^2 (k \in N) \\ \mu''_n(t), & \text{otherwise,} \end{cases}$$

where

$$\mu'_n(t) = \begin{cases} 1 + t - 2n, & \text{if } t \in [2n-1, 2n] \\ 1 - t + 2n, & \text{if } t \in [2n, 2n+1] \\ 0, & \text{otherwise,} \end{cases}$$

and

$$\mu''_n(t) = \begin{cases} \frac{n}{3}(t-2) + 1, & \text{if } t \in [\frac{2n-3}{n}, 2) \\ -\frac{n}{3}(t-2) + 1, & \text{if } t \in [2, \frac{2n+3}{2}] \\ 0, & \text{otherwise.} \end{cases}$$

It can be easily shown that (X_n) is not rough convergent, for any $r \geq 0$, but it is rough statistically convergent for $r \geq 0.5$.

THEOREM 2.3. *If (X_n) and (Y_n) are two sequences in $\left(L(\mathbb{R}), \bar{d}\right)$ such that $Y_n \xrightarrow{r_1 st} X_*$ and $\delta\left(\left\{n \in N : \bar{d}(X_n, Y_n) \leq r_2\right\}\right) = 1$, then (X_n) is $(r_1 + r_2)$-statistically convergent to X_*, for $r_1 \geq 0$ and $r_2 \geq 0$.*

Proof. Since $Y_n \xrightarrow{r_1 st} X_*$, we have, for all $\varepsilon > 0$,

$$\delta\left(\left\{n \in N : \bar{d}(X_n, X_*) \geq r_1 + \varepsilon\right\}\right) = 0,$$

i.e. $\delta(A) = 1$, where $A = \left\{n \in N : \bar{d}(Y_n, X_*) < r_1 + \varepsilon\right\}$.

Let $B = \left\{n \in N : \bar{d}(X_n, Y_n) \leq r_2\right\}$. It is given that $\delta(B) = 1$. Then $\delta(A \cap B) = 1$. So, for all such $n \in A \cap B$, we have

$$\bar{d}(X_n, X_*) \leq \bar{d}(X_n, Y_n) + \bar{d}(Y_n, X_*) < r_1 + r_2 + \varepsilon,$$

for $n \geq k$. Hence the proof is complete. □

In particular, for $r_1 = 0$ and $r_2 = r > 0$, we get an approximate sequence (X_n) of a statistically convergent sequence $Y_n \to X_*$ with

$$\delta\left(\left\{n \in N : \bar{d}(X_n, Y_n) \leq r\right\}\right) = 1,$$

which is r-statistically convergent to X_*.

THEOREM 2.4. *If (X_n) and (Y_n) are two sequences in $\left(L(\mathbb{R}), \bar{d}\right)$ such that $X_n \overset{r_1 st}{\longrightarrow} X_*$ and $Y_n \overset{r_2 st}{\longrightarrow} Y_*$, then $X_n + Y_n \overset{(r_1+r_2)st}{\longrightarrow} X_* + Y_*$, for $r_1 \geq 0$ and $r_2 \geq 0$.*

Proof. Let $X_n \overset{r_1 st}{\longrightarrow} X_*$ and $Y_n \overset{r_2 st}{\longrightarrow} Y_*$. Then we have $\delta(A_1) = 1$ and $\delta(A_2) = 1$, where $A_1 = \left\{n \in N : \bar{d}(X_n, X_*) < r_1 + \frac{\varepsilon}{2}\right\}$ and $A_2 = \{n \in N : \bar{d}(Y_n, Y_*) < r_2 + \frac{\varepsilon}{2}\}$. Therefore $\delta(A_1 \cap A_2) = 1$. So, for all such $n \in A_1 \cap A_2$, we have $\bar{d}\left(X_n + Y_n, X_* + Y_*\right) \leq \bar{d}(X_n, X_*) + \bar{d}(Y_n, Y_*) < (r_1 + r_2) + \varepsilon$ for $n \geq k$. Hence the proof follows. □

In view of the existing techniques, we state the following results without proof.

THEOREM 2.5. *For any $c \in \mathbb{R}$, if $X_n \overset{rst}{\longrightarrow} X_*$, then $cX_n \overset{|c|rst}{\longrightarrow} cX_*$.*

It is known that the limit of a statistically convergent sequence of fuzzy numbers has an unique limit point. But this property is not maintained in the case of rough statistical convergence with roughness degree $r > 0$. So, in the case of rough statistical convergence we get an r-statistical limit set. We discuss some basic properties of the r-statistical limit set of a sequence of fuzzy numbers.

DEFINITION 2.6. Let X_* be an r- statistical limit point of (X_n), which is not necessarily unique. Consider the r-statistical limit set of (X_n), defined by $st - LIM^r X_n = \left\{X_* \in L(\mathbb{R}) : X_n \overset{rst}{\to} X_*\right\}$, i.e. $st - LIM^r X_n = \{X_* \in L(\mathbb{R}) : [X_*^\alpha] \subseteq [st\, lim \sup(X_{ln}^\alpha) - r, st\, lim \inf(X_{un}^\alpha) + r]\}$, where $[X_n^\alpha] = [X_{ln}^\alpha, X_{un}^\alpha]$

If $st - LIM^r X_n = \emptyset$, for any sequence (X_n) of fuzzy numbers, then (X_n) is not r- statistically convergent, for any $r \geq 0$.

EXAMPLE 2.7. Let $X_n(t) = \begin{cases} 1 + t - 2n, & \text{if } t \in [2n-1, 2n] \\ 1 - t + 2n, & \text{if } t \in [2n, 2n+1] \\ 0, & \text{otherwise.} \end{cases}$

Here (X_n) is not r-statistically convergent, since $st - LIM^r X_n = \emptyset$, for any $r \geq 0$.

PROPOSITION 2.8. *The set* $st - LIM^r X_n$ *of an arbitrary sequence of fuzzy numbers* (X_n) *of* $\left(L(\mathbb{R}), \bar{d}\right)$ *is a closed set.*

Proof. If $st - LIM^r X_n = \emptyset$, then the hypothesis is true. Suppose that $st - LIM^r X_n \neq \emptyset$. Let (Y_n) be a sequence in $st - LIM^r X_n$ which converges to Y. We show that $Y \in \overline{LIM^r X_n}$. Since (Y_n) converges to Y, we have $\bar{d}(Y_n, Y) < \frac{\epsilon}{2}$, $\forall n \geq n_0$. In particular, $\bar{d}(Y_{n_0}, Y) < \frac{\epsilon}{2}$ and, by the definition of $st{-}LIM^r X_n$, we have, $\delta\left(\left\{n \in N : \bar{d}(X_n, Y_{n_0}) \geq r + \frac{\epsilon}{2}\right\}\right) = 0$. Now, for all $n \in \left\{n \in N : \bar{d}(X_n, Y_{n_0}) < r + \frac{\epsilon}{2}\right\}$, we have $\bar{d}(X_n, Y) \leq \bar{d}(X_n, Y_{n_0}) + \bar{d}(Y_{n_0}, Y) < r + \epsilon$. So $\left\{n \in N : \bar{d}(X_n, Y) < r + \varepsilon\right\} \supseteq \left\{n \in N : \bar{d}(X_n, Y_{n_0}) < r + \frac{\epsilon}{2}\right\}$, i.e. $\delta\left\{n \in N : \bar{d}(X_n, Y) \geq r + \varepsilon\right\} = 0$, which completes the proof. □

PROPOSITION 2.9. *For any fuzzy number sequence* (X_n) *in* $\left(L(\mathbb{R}), \bar{d}\right)$*, we have* $\operatorname{diam}(st - LIM^r X_n) \leq 2r$.

Proof. If possible, let $\operatorname{diam}(st - LIM^r X_n) > 2r$. Then there exist $Y, Z \in st - LIM^r X_n$ such that $\bar{d}(Y, Z) = d_1 > 2r$. Let $\varepsilon = \frac{d_1 - 2r}{2}$. Since $Y, Z \in st - LIM^r X_n$, we have $\delta(K_1) = 0$ and $\delta(K_2) = 0$, where $K_1 = \{n \in N : \bar{d}(X_n, Y) \geq r + \varepsilon\}$, $K_2 = \left\{n \in N : \bar{d}(X_n, Z) \geq r + \varepsilon\right\}$. Hence, for all $n \in (K_1^c \cap K_2^c)$, we have $\bar{d}(Y, Z) \leq \bar{d}(X_n, Y) + \bar{d}(X_n, Z) < 2(r + \epsilon) = d_1$, $\forall n \geq n_0$, which is a contradiction. Therefore $\operatorname{diam}(LIM^r X_n) \leq 2r$. □

PROPOSITION 2.10. *Let* (X_n) *be statistically convergent to* X_**. Then* $st - LIM^r X_n = \bar{B}_r(X_*)$.

Proof. Since $X_n \xrightarrow{st} X_*$, we have $\delta\left\{n \in N : \bar{d}(X_n, X_*) \geq \varepsilon\right\} = 0$. Let $Y \in \bar{B}_r(X_*) = \left\{Y \in L(\mathbb{R}) : \bar{d}(Y, X_*) \leq r\right\}$. Now, for all $n \in N$ with $\bar{d}(X_n, X_*) < \varepsilon$, we have $\bar{d}(X_n, Y) \leq \bar{d}(X_n, X_*) + \bar{d}(Y, X_*) < r + \varepsilon$.

Since $\delta\left(\left\{n \in N : \bar{d}(X_n, X_*) < \varepsilon\right\}\right) = 1$ and $\left\{n \in N : \bar{d}(X_n, Y) < r + \varepsilon\right\} \supseteq \left\{n \in N : \bar{d}(X_n, X_*) < \varepsilon\right\}$, we have $Y \in st - LIM^r X_n$. Consequently, we can write $st - LIM^r X_n = \bar{B}_r(X_*)$. □

PROPOSITION 2.11. *For all* $r > 0$*, a statistically bounded sequence* (X_n) *of fuzzy numbers always contains a subsequence* (X_{n_k}) *with* $st{-}LIM^{(X_{n_k}),r} X_{n_k} \neq \emptyset$.

Proof. It is known that every statistically bounded sequence has a statistically convergent subsequence, so (X_n) contains a statistically convergent subsequence (X_{n_k}). Let X_* be the limit point of (X_{n_k}). Then $st - LIM^r X_{n_k} = \bar{B}_r(X_*)$ and, for $r > 0$, $st - LIM^{(X_{n_k}),r} X_{n_k} = \left\{X_{n_k} : \bar{d}(X_{n_k}, X_*) \leq r\right\} \neq \emptyset$. □

PROPOSITION 2.12. *If* (X'_n) *is a non-thin subsequence of* (X_n)*, then* $st - LIM^r X_n \subseteq st - LIM^r X'_n$.

Proof. The proof is obvious. □

Proposition 2.13. *For an arbitrary* $C \in \Gamma_X$ *of a sequence* $X = (X_n)$*, we have* $\bar{d}(X_*, C) \le r$*, for all* $X_* \in st - LIM^r X$.

Proof. Since $C \in \Gamma_X$, $\delta(A) \neq 0$, where $A = \{n \in N : \bar{d}(X_n, C) < \frac{\varepsilon}{2}\}$. Let $X_* \in st - LIM^r X$. Then, for all $\varepsilon > 0$, $\delta(B) = 1$, where $B = \{n \in N : \bar{d}(X_n, X_*) < r + \frac{\varepsilon}{2}\}$. Therefore, for all $n \in A \cap B$, we have $\bar{d}(X_*, C) \le \bar{d}(X_n, X_*) + \bar{d}(X_n, C) < r + \epsilon$. Hence the proof follows. □

Proposition 2.14. (*a*) *If* $C \in \Gamma_X$*, then* $st - LIM^r X \subseteq B_r(C)$.
(*b*) $st - LIM^r X = \bigcap_{C \in \Gamma_X} \bar{B}_r(C) = \{Y_* \in L(R) : \Gamma_X \subseteq \bar{B}_r(Y_*)\}$.

Proof. (a) Let $X_* \in LIM^r X_n$ and $C \in \Gamma_X$. Then $\bar{d}(X_*, C) \le r$, i.e. $st - LIM^r X \subseteq B_r(C)$.
(b) From the above theorem, we have

$$st - LIM^r X \subseteq \cap B_r(C). \tag{1}$$

Let $Y \in \bigcap_{C \in \Gamma_X} B_r(C)$. Then $\bar{d}(Y, C) \le r$, $\forall C \in \Gamma_X$, which is equivalent to $\Gamma_X \subseteq \bar{B}_r(Y)$, i.e.

$$\bigcap_{C \in \Gamma_X} B_r(C) \subseteq \{Y_* \in L(\mathbb{R}) : \Gamma_X \subseteq \bar{B}_r(Y_*)\}. \tag{2}$$

Now, let $Y \notin st - LIM^r X$. Then there exists an $\varepsilon > 0$ such that $\delta(\{n \in N : \bar{d}(X_n, Y) \ge r + \varepsilon\}) \neq 0$, which implies the existence of a statistical cluster point C of the sequence X that satisfies $\bar{d}(Y, C) > r + \frac{\varepsilon}{2}$, i.e. Γ_X is not a subset of $\bar{B}_r(Y)$ and $Y \notin \{Y_* \in L(\mathbb{R}) : \Gamma_X \subseteq \bar{B}_r(Y_*)\}$.
Therefore,

$$\{Y_* \in L(\mathbb{R}) : \Gamma_X \subseteq \bar{B}_r(Y_*)\} \subseteq st - LIM^r X \tag{3}$$

The proof follows, by following (1), (2) and (3). □

Proposition 2.15. *Let* $X = (X_n)$ *be a statistically bounded sequence of fuzzy numbers. If* $r = \text{diam}(\Gamma_X)$*, then* $\Gamma_X \subseteq st - LIM^r X$.

Proof. Take $C \notin st - LIM^r X$. Then there exists $\varepsilon > 0$ such that $\delta(\{n \in N : \bar{d}(X_n, C) \ge r + \varepsilon\}) \neq 0$. Since the sequence is statistically bounded and in view of the above inequality, there exists another statistical cluster point C' such that $\bar{d}(C, C') > r + \frac{\varepsilon}{2}$. So, we get $\text{diam}(\Gamma_X) > r + \frac{\varepsilon}{2}$, which is a contradiction. Hence the result follows. □

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