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CERTAIN CLASS OF ANALYTIC FUNCTIONS WITH VARYING ARGUMENTS DEFINED BY SĂLĂGEAN AND RUSCHEWEYH DERIVATIVE

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Abstract. In this paper we derive some results for a certain new class of analytic functions with varying arguments defined by using Sălăgean and Ruscheweyh derivative.

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1. INTRODUCTION

Let $\mathcal{A}$ denote the class of functions of the form

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k, \tag{1}$$

which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$.

DEFINITION 1.1 ([2]). For $f \in \mathcal{A}$, $\lambda \geq 0$ and $n \in \mathbb{N}$, the operator $\mathscr{D}_\lambda^n \colon \mathcal{A} \to \mathcal{A}$ is defined by

$$\mathscr{D}_\lambda^0 f(z) = f(z),$$
$$\mathscr{D}_\lambda^1 f(z) = (1-\lambda) f(z) + \lambda z f'(z) = \mathscr{D}_\lambda f(z), \ldots,$$
$$\mathscr{D}_\lambda^{n+1} f(z) = (1-\lambda) \mathscr{D}_\lambda^n f(z) + \lambda z \left(\mathscr{D}_\lambda^n f(z)\right)' = \mathscr{D}_\lambda \left(\mathscr{D}_\lambda^n f(z)\right), \, z \in U.$$

REMARK 1.2 ([7]). For $\lambda = 1$ in the above definition we obtain the Sălăgean differential operator.

DEFINITION 1.3 ([6]). For $f \in \mathcal{A}, n \in \mathbb{N}$, the operator $\mathscr{R}^n : \mathcal{A} \to \mathcal{A}$ is defined by

$$\mathscr{R}^0 f(z) = f(z), \ldots,$$
$$(n+1) \mathscr{R}^{n+1} f(z) = z \left(\mathscr{R}^n f(z)\right)' + n \mathscr{R}^n f(z), \, z \in U.$$

DEFINITION 1.4. Let $\gamma, \lambda \geq 0$ and $n \in \mathbb{N}$. Let $\mathscr{L}^n : \mathcal{A} \to \mathcal{A}$ be the operator given by

$$\mathscr{L}^n f(z) = (1-\gamma) \mathscr{R}^n f(z) + \gamma \mathscr{D}_\lambda^n f(z), \, z \in U.$$

REMARK 1.5. If $f \in \mathcal{A}$ and $f(z) = z + \sum_{k=2}^{\infty} a_k z^k$, then

$$\mathscr{L}^n f(z) = z + \sum_{k=2}^{\infty} \left\{ \gamma \left[1 + (k-1)\lambda\right]^n + (1-\gamma) \frac{(n+k-1)!}{n!\,(k-1)!} \right\} a_k z^k, \, z \in U.$$

Definition 1.6 ([4]). Let f and g be analytic functions in U. We say that the function f is *subordinate* to the function g, if there exists an analytic function w on U with $w(0) = 0$ and $|w(z)| < 1$, $z \in U$, such that $f(z) = g(w(z))$, for all $z \in U$. We denote by $\prec$ the subordination relation.

Definition 1.7. For $\widetilde{\lambda} \geq 0$, $-1 \leq A < B \leq 1$, $0 < B \leq 1$ and $n \in \mathbb{N}_0$, let $L(n, \widetilde{\lambda}, A, B)$ denote the subclass of $\mathcal{A}$ which contains the functions $f = f(z)$ of the form (1) that satisfy

$$(1 - \widetilde{\lambda})(\mathscr{L}^n f(z))' + \widetilde{\lambda}(\mathscr{L}^{n+1} f(z))' \prec \frac{1 + Az}{1 + Bz}. \tag{2}$$

Attiya and Aouf defined in [3] the class $\mathscr{R}(n, \lambda, A, B)$, using a condition similar to (2), where, instead of the operator $\mathscr{L}^n$, they used the Ruscheweyh operator.

Definition 1.8 ([9]). A function $f = f(z)$ of the form (1) is said to be in the class $V(\theta_k)$ if $f \in \mathcal{A}$ and $arg(a_k) = \theta_k$, for all $k \geq 2$. If there exists $\delta \in \mathbb{R}$ such that $\theta_k + (k - 1)\delta \equiv \pi(mod\ 2\pi)$, for all $k \geq 2$, then f is said to be in the class $V(\theta_k, \delta)$. The union of $V(\theta_k, \delta)$, taken over all possible sequences $\{\theta_k\}$ and all possible real numbers δ, is denoted by V.

Let $VL(n, \widetilde{\lambda}, A, B)$ denote the subclass of V consisting of functions $f \in L(n, \widetilde{\lambda}, A, B)$.

2. MAIN RESULTS

2.1. Coefficient estimates

Theorem 2.1. *Let the function $f = f(z)$ given by* (1) *be in* V*. Then* $f = f(z) \in VL(n, \widetilde{\lambda}, A, B)$ *if and only if*

$$T(f) = \sum_{k=2}^{\infty} kC_k (1 + B) |a_k| \leq B - A, \tag{3}$$

where

$$C_k = \gamma [1 + (k-1)\lambda]^n \left[1 + \widetilde{\lambda}\lambda(k-1)\right] + \frac{(n+k-1)!}{n!\,(k-1)!} (1 - \gamma) \left[1 + \widetilde{\lambda}\frac{k-1}{n+1}\right].$$

Moreover, the extremal functions for (3) *are*

$$f(z) = z + \frac{B - A}{kC_k (1 + B)} \mathrm{e}^{\mathrm{i}\theta_k} z^k, \ k \geq 2.$$

Proof. Our arguments are based on the technique used in [5].

Suppose that $f = f(z) \in VL(n, \widetilde{\lambda}, A, B)$. Then

$$h(z) = (1 - \widetilde{\lambda})(\mathscr{L}^n f(z))' + \widetilde{\lambda}(\mathscr{L}^{n+1} f(z))' = \frac{1 + Aw(z)}{1 + Bw(z)}, \tag{4}$$

where $w \in H = \{w \text{ analytic}, w(0) = 0 \text{ and } |w(z)| < 1, z \in U\}$. From this we have $w(z) = \frac{1-h(z)}{Bh(z)-A}$. Therefore

$$h(z) = 1 + \sum_{k=2}^{\infty} \left\{ \gamma \left[1 + (k-1)\lambda\right]^n \left[1 + \widetilde{\lambda}(1-k)\right] + \frac{(n+k-1)!}{n!\,(k-1)!}(1-\gamma)\left[1 + \widetilde{\lambda}\frac{k-1}{n+1}\right] \right\} k a_k z^{k-1}.$$

Hence $h(z) = 1 + \sum_{k=2}^{\infty} C_k k a_k z^{k-1}$ and thus $|w(z)| < 1$ implies

$$\left| \frac{\sum_{k=2}^{\infty} C_k k a_k z^{k-1}}{(B-A) + B\sum_{k=2}^{\infty} C_k k a_k z^{k-1}} \right| < 1. \tag{5}$$

Since $f = f(z) \in V$, $f = f(z)$ lies in $V(\theta_k, \delta)$ for some sequence $\{\theta_k\}$ and a real number δ with $\theta_k + (k-1)\delta \equiv \pi (\text{mod } 2\pi)$, for all $k \geq 2$.

Set $z = re^{\mathrm{i}\delta}$ in (5). Then $a_k z^{k-1} = |a_k| r^{k-1}$ and

$$\left| \frac{\sum_{k=2}^{\infty} C_k k |a_k| r^{k-1}}{(B-A) - B\sum_{k=2}^{\infty} C_k k |a_k| r^{k-1}} \right| < 1. \tag{6}$$

Since $\mathrm{Re}\{w(z)\} < |w(z)| < 1$, we have

$$\mathrm{Re}\left\{ \frac{\sum_{k=2}^{\infty} C_k k |a_k| r^{k-1}}{(B-A) - B\sum_{k=2}^{\infty} C_k k |a_k| r^{k-1}} \right\} < 1. \tag{7}$$

So

$$\sum_{k=2}^{\infty} k C_k (1+B) |a_k| r^{k-1} \leq B - A. \tag{8}$$

Letting $r \to 1$, we get $\sum_{k=2}^{\infty} k C_k (1+B) |a_k| \leq B - A$.

Conversely, assume that $f = f(z) \in V$ satisfies (3). Since $r^{k-1} < 1$, we have

$$\begin{aligned} \left| \sum_{k=2}^{\infty} k |a_k| z^{k-1} C_k \right| &\leq \sum_{k=2}^{\infty} k |a_k| r^{k-1} C_k \\ &\leq (B-A) - B\sum_{k=2}^{\infty} k |a_k| r^{k-1} C_k \\ &\leq \left| (B-A) + B\sum_{k=2}^{\infty} k a_k z^{k-1} C_k \right|, \end{aligned}$$

which gives (5) and hence it follows that

$$(1-\widetilde{\lambda})(\mathscr{L}^n f(z))' + \widetilde{\lambda}(\mathscr{L}^{n+1} f(z))' = \frac{1+Aw(z)}{1+Bw(z)},$$

that is $f = f(z) \in VL(n, \widetilde{\lambda}, A, B)$.

□

Corollary 2.2. *Let the function* $f = f(z)$ *given by* (1) *be in the class* $VL(n, \widetilde{\lambda}, A, B)$. *Then*

$$|a_k| \leq \frac{B-A}{kC_k\,(1+B)},\ k \geq 2.$$

The result given by (3) *is sharp for the functions*

$$f(z) = z + \frac{B-A}{kC_k\,(1+B)} \mathrm{e}^{\mathrm{i}\theta_k} z^k,\ k \geq 2.$$

2.2. Distortion theorems

Theorem 2.3. *Let the function* $f = f(z)$ *given by* (1) *be in the class* $VL(n, \widetilde{\lambda}, A, B)$. *Then*

$$|z| - \frac{B-A}{2C_2\,(1+B)}\,|z|^2 \leq |f(z)| \leq |z| + \frac{B-A}{2C_2\,(1+B)}\,|z|^2\,. \tag{9}$$

Proof. Our arguments are based on the technique used by Silverman [9]. Let

$$\Phi\,(k) = kC_k\,(1+B)\,. \tag{10}$$

Then Φ is an increasing function with respect to k $(k \geq 2)$ and thus

$$\Phi(2)\sum_{k=2}^{\infty}|a_k| \leq \sum_{k=2}^{\infty}\Phi(k)\,|a_k| \leq B - A,$$

or, equivalently,

$$\sum_{k=2}^{\infty}|a_k| \leq \frac{B-A}{\Phi(2)} = \frac{B-A}{2C_2\,(1+B)}. \tag{11}$$

Hence, we have

$$|f(z)| \leq |z| + \sum_{k=2}^{\infty}|a_k|\,|z|^k \leq |z| + |z|^2\sum_{k=2}^{\infty}|a_k|$$

and thus

$$|f(z)| \leq |z| + \frac{B-A}{2C_2\,(1+B)}\,|z|^2\,.$$

Also, we have

$$|f(z)| \geq |z| - \sum_{k=2}^{\infty}|a_k|\,|z|^k \geq |z| - |z|^2\sum_{k=2}^{\infty}|a_k|\,.$$

Therefore

$$|f(z)| \geq |z| - \frac{B-A}{2C_2\,(1+B)}\,|z|^2\,.$$

The result is sharp for the function

$$f(z) = z + \frac{B-A}{2C_2\,(1+B)}\mathrm{e}^{\mathrm{i}\theta_2} z^2,$$

at $z = \pm |z| \, \mathrm{e}^{-\mathrm{i}\theta_2}$.

□

COROLLARY 2.4. *Let the function $f = f(z)$ given by* (1) *be in the class $VL(n, \widetilde{\lambda}, A, B)$. Then $f(z) \in U(0, r_1)$, where $r_1 = 1 + \frac{B-A}{2C_2(1+B)}$.*

THEOREM 2.5. *Let the function $f = f(z)$ given by* (1) *be in the class $VL(n, \widetilde{\lambda}, A, B)$. Then*

$$1 - \frac{B-A}{C_2\,(1+B)}\,|z| \le \left|f'(z)\right| \le 1 + \frac{B-A}{C_2\,(1+B)}\,|z|\,. \tag{12}$$

The result is sharp.

Proof. Let $\frac{\Phi(k)}{k} = C_k\,(1+B)$. This is an increasing function with respect to k $(k \ge 2)$. According to Theorem 2.1, we have

$$\frac{\Phi(2)}{2}\sum_{k=2}^{\infty} k\,|a_k| \le \sum_{k=2}^{\infty}\Phi(k)\,|a_k| \le B - A,$$

or equivalently

$$\sum_{k=2}^{\infty} k\,|a_k| \le \frac{B-A}{\Phi(2)} = \frac{B-A}{C_2\,(1+B)}.$$

Hence, we have

$$\left|f'(z)\right| \le 1 + |z|\sum_{k=2}^{\infty} k\,|a_k| \le 1 + \frac{B-A}{C_2\,(1+B)}\,|z|\,.$$

So

$$\left|f'(z)\right| \ge 1 - |z|\sum_{k=2}^{\infty} k\,|a_k| \ge 1 - \frac{B-A}{C_2\,(1+B)}\,|z|\,.$$

□

COROLLARY 2.6. *Let the function $f = f(z)$ given by* (1) *be in the class $VL(n, \widetilde{\lambda}, A, B)$. Then $f'(z) \in U(0, r_2)$, where $r_2 = 1 + \frac{B-A}{C_2(1+B)}$.*

2.3. EXTREME POINTS

THEOREM 2.7. *Let the function $f = f(z)$ given by* (1) *be in the class $VL(n, \widetilde{\lambda}, A, B)$, with $arg(a_k) = \theta_k$, where $\theta_k + (k-1)\delta \equiv \pi (\mathrm{mod}\ 2\pi)$, for all $k \ge 2$. Define*

$$f_1(z) = z$$

and

$$f_k(z) = z + \frac{B-A}{kC_k\,(1+B)}\mathrm{e}^{\mathrm{i}\theta_k} z^k,\ k \ge 2, z \in U.$$

Then $f = f(z) \in VL(n, \widetilde{\lambda}, A, B)$ if and only if $f = f(z)$ can be expressed by $f(z) = \sum\limits_{k=1}^{\infty} \mu_k f_k(z)$, where $\mu_k \ge 0$ and $\sum\limits_{k=1}^{\infty} \mu_k = 1$.

Proof. If $f(z)=\sum\limits_{k=1}^{\infty}\mu_k f_k(z)$, $\mu_k\geq 0$, and $\sum\limits_{k=1}^{\infty}\mu_k=1$, then

$$\sum_{k=2}^{\infty}kC_k\left(1+B\right)\frac{B-A}{kC_k\left(1+B\right)}\mu_k=\sum_{k=2}^{\infty}(B-A)\mu_k$$
$$=(1-\mu_1)(B-A)\leq B-A.$$

Hence $f=f(z)\in VL(n,\widetilde{\lambda},A,B)$. Conversely, let the function $f=f(z)$ given by (1) be in the class $VL(n,\widetilde{\lambda},A,B)$ and define

$$\mu_k=\frac{kC_k\left(1+B\right)}{B-A}\left|a_k\right|,\ k\geq 2,$$

and

$$\mu_1=1-\sum_{k=2}^{\infty}\mu_k.$$

From Theorem 2.1, $\sum\limits_{k=2}^{\infty}\mu_k\leq 1$ and so $\mu_1\geq 0$. Since $\mu_k f_k(z)=\mu_k z+a_k z^k$, for $k\geq 2$, we obtain

$$\sum_{k=1}^{\infty}\mu_k f_k(z)=z+\sum_{k=2}^{\infty}a_k z^k=f(z).$$

□

Remark. The operator I_c in the following theorem is the well-known Bernardi operator, see [8].

Theorem 2.8. *Let*

$$F(z)=I_cf(z)=\frac{c+1}{z^c}\int_0^z f(t)t^{c-1}\mathrm{d}t,\ c>-1.$$

If $f\in VL(n,\widetilde{\lambda},A,B)$, then $F\in VL(n,\widetilde{\lambda},A^*,B)$, where $A^*=\frac{B+A(c+1)}{c+2}>A$. The result is sharp.

Proof. Let $f\in VL(n,\widetilde{\lambda},A,B)$ and suppose it has the form (1). Then

$$F(z)=\frac{c+1}{z^c}\int_0^z\left(t+\sum_{k=2}^{\infty}a_k t^k\right)t^{c-1}\mathrm{d}t$$
$$=z+\sum_{k=2}^{\infty}\frac{c+1}{c+k}a_k z^k=z+\sum_{k=2}^{\infty}b_k z^k.$$

Since $f\in VL(n,\widetilde{\lambda},A,B)$, we have $\sum_{k=2}^{\infty}kC_k\left(1+B\right)\left|a_k\right|\leq B-A$ or, equivalently,

$$\frac{\sum\limits_{k=2}^{\infty}kC_k\left(1+B\right)\left|a_k\right|}{B-A}\leq 1.$$

We know, from Theorem 2.1, that $F \in VL(n, \widetilde{\lambda}, A^*, B)$ if and only if

$$\frac{\sum\limits_{k=2}^{\infty} kC_k\,(1+B)\,\dfrac{c+1}{c+k}\,|a_k|}{B-A^*} \le 1. \tag{13}$$

Next, we show that

$$\frac{kC_k\,(1+B)\,\frac{c+1}{c+k}\,|a_k|}{B-A^*} \le \frac{kC_k\,(1+B)\,|a_k|}{B-A},\ k \ge 2, \tag{14}$$

and we note that (14) implies (13). The inequalities in (14) follow from

$$\frac{c+1}{(c+k)(B-A^*)} \le \frac{1}{B-A},$$

$$(c+1)(B-A) \le (c+k)(B-A^*),\ k \ge 2,$$

$$A^* \le \frac{B(k-1)+A(c+1)}{(c+k)},\ k \ge 2.$$

Let us consider the function given by $E(x) = \frac{B(x-1)+A(c+1)}{x+c}$. Its derivative is $E'(x) = \frac{(B-A)(c+1)}{(x+c)^2} > 0$, hence $E = E(x)$ is an increasing function. For our case, we need $A^* \le E(k)$, for all $k \ge 2$. For this reason, we choose $A^* = E(2) = \frac{B+A(c+1)}{c+2}$. We note that $A^* > A$, because

$$B + A(c+1) > A(c+2) \Leftrightarrow B > A.$$

The result is sharp, because, if $f_2(z) = z + \frac{B-A}{2C_2(1+B)} e^{i\theta_2} z^2$, then $F_2 = I_c f_2$ belongs to $VL(n, \widetilde{\lambda}, A^*, B)$ and its coefficients satisfy the corresponding inequality in (3) with equality. Indeed, we have

$$F_2(z) = z + \frac{B-A}{2C_2\,(1+B)}\frac{c+1}{c+2} e^{i\theta_2} z^2 = z + \frac{B-A^*}{2C_2\,(1+B)} e^{i\theta_2} z^2$$

and

$$T(F_2) = 2C_2\,(1+B)\,\frac{B-A^*}{2C_2\,(1+B)} = B - A^*.$$

□

If $A = 2\alpha - 1, A^* = 2\beta - 1$, then, from Theorem 2.8, we get the following particular case.

Corollary 2.9. *If* $f \in VL(n, \widetilde{\lambda}, 2\alpha - 1, B)$*, then* $F \in VL(n, \widetilde{\lambda}, 2\beta - 1, B)$*, where*

$$\beta = \beta(\alpha) = \frac{B + 1 + 2\alpha(c+1)}{2(c+2)} \ge \alpha.$$

The result is sharp.

Theorem 2.10. *If* $f \in VL(n, \widetilde{\lambda}, A, B)$*, then* $F \in VL(n, \widetilde{\lambda}, A, B^*)$*,where*

$$B^* = \frac{A\,(1+B)\,(c+2) + (B-A)\,(c+1)}{(1+B)\,(c+2) - (B-A)\,(c+1)} < B.$$

The result is sharp.

Proof. Let $f \in VL(n, \widetilde{\lambda}, A, B)$ and suppose it has the form (1). Since $f \in VL(n, \widetilde{\lambda}, A, B)$, we have $\sum_{k=2}^{\infty} kC_k\,|a_k| \le B - A$ or, equivalently,

$$\frac{\sum\limits_{k=2}^{\infty} kC_k\,(1+B)\,|a_k|}{B-A} \le 1.$$

We know, from Theorem 2.1, that $F \in VL(n, \widetilde{\lambda}, A, B^*)$ if and only if

$$\sum_{k=2}^{\infty} kC_k\,(1+B^*)\,|b_k| \le B^* - A$$

or

$$\frac{\sum\limits_{k=2}^{\infty} kC_k\,(1+B^*)\,\frac{c+1}{c+k}\,|a_k|}{B^*-A} \le 1. \tag{15}$$

We note that

$$\frac{kC_k\,(1+B^*)\,\frac{c+1}{c+k}\,|a_k|}{B^*-A} \le \frac{kC_k\,(1+B)\,|a_k|}{B-A},\ k \ge 2 \tag{16}$$

implies (15). The inequalities in (16) follow from

$$\frac{(c+1)\,(1+B^*)}{(c+k)(B^*-A)} \le \frac{1+B}{B-A},$$

$$\frac{A\,(1+B)\,(c+k) + (B-A)\,(c+1)}{(1+B)\,(c+k) - (B-A)\,(c+1)} \le B^*,\ k \ge 2.$$

Let

$$E(x) = \frac{A\,(1+B)\,(c+x) + (B-A)\,(c+1)}{(1+B)\,(c+x) - (B-A)\,(c+1)}.$$

Its derivative is

$$E'(x) = \frac{-\,(A+1)\,(c+1)\,(B-A)\,(1+B)}{\left[(1+B)\,(c+x) - (B-A)\,(c+1)\right]^2} < 0.$$

Hence $E = E(x)$ is a decreasing function. For our case, we need $E(k) \le B^*$. For this reason, we choose

$$B^* = E(2) = \frac{A\,(1+B)\,(c+2) + (B-A)\,(c+1)}{(1+B)\,(c+2) - (B-A)\,(c+1)}$$

and we note that

$B^* < B \Leftrightarrow (B-A)\,(c+1)\,(1+B) < (1+B)\,(c+2)\,(B-A) \Leftrightarrow c+1 < c+2.$

The result is sharp, because, if

$$f_2(z) = z + \frac{B-A}{2C_2\,(1+B)} \mathrm{e}^{\mathrm{i}\theta_2} z^2,$$

then $F_2 = I_c f_2$ belongs to $VL(n, \widetilde{\lambda}, A, B^*)$ and its coefficients satisfy the corresponding inequality in (3) with equality. Indeed, we have

$$F_2(z) = z + \frac{B-A}{2C_2\,(1+B)} \frac{c+1}{c+2} \mathrm{e}^{\mathrm{i}\theta_2} z^2 = z + \frac{B^*-A}{2C_2\,(1+B^*)} \mathrm{e}^{\mathrm{i}\theta_2} z^2$$

and

$$T(F_2) = 2C_2\,(1+B^*)\,\frac{B^*-A}{2C_2\,(1+B^*)} = B^* - A.$$

□

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