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ON THE STRUCTURE OF NEAT-INJECTIVE ENVELOPES

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Abstract. We study the neat injective envelope for abelian groups and give some results about its structure in terms of the basic subgroups.
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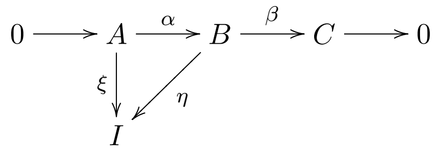
1. INTRODUCTION

It is well known that every abelian group A can be embedded in a minimal injective (i.e. divisible) group which is called an injective envelope of A (see [1]). Similar results were proved for pure-injective envelope [1] and then for neat-injective envelopes (see [2]). Our main purpose in this paper is to describe neat-injective envelopes of an abelian group A in terms of the basic subgroups $B_p(A)$ of p-component $T_p(A)$ of A and $A/T(A)$. We will be considering only abelian groups and use notations and some well known facts from [1]. P will denote the set of all prime integers, $A \trianglelefteq B$ means that A is an essential subgroup of B.

2. PRELIMINARIES

A subgroup H of G is said to be a *neat subgroup* of G, if any equation $px = a$ with $a \in H$ is solvable in H, whenever it is solvable in G for every prime integer p. Equivalently $pH = H \cap pG$ for every prime p. A short exact sequence $E : 0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$ is said to be neat exact, if $Im\,\alpha$ is a neat subgroup of A. The subgroup of $\mathrm{Ext}(C, A)$ consisting of all neat short exact sequences is denoted by $\mathrm{Next}(C, A)$. I is called a *neat-injective group* if for every neat exact sequence $E : 0 \longrightarrow A \xrightarrow{\alpha} B \xrightarrow{\beta} C \longrightarrow 0$ and homomorphism $\xi : A \longrightarrow I$ there exists $\eta : B \longrightarrow I$ such that $\eta \circ \alpha = \xi$,



The following Lemma from [5] describes the structure of neat injective groups.

LEMMA 1. *I is neat injective if and only if* $I = D \oplus \prod_p T_p$, *where D is divisible,* $pT_p = 0$, *and p ranges over all primes.*

The following Lemma, the proof of which is very easy, will be referred in proving the main results.

Lemma 2. *If* $\{E_i : i \in I\}$ *is a family of short neat-exact sequences*

$$E_i : \quad 0 \longrightarrow A_i \longrightarrow B_i \longrightarrow C_i \longrightarrow 0,$$

then their direct sum

$$\bigoplus_{i\in I} E_i : \quad 0 \longrightarrow \bigoplus_{i\in I} A_i \longrightarrow \bigoplus_{i\in I} B_i \longrightarrow \bigoplus_{i\in I} C_i \longrightarrow 0$$

is also neat exact.

The following result is an immediate consequence of Exercise 10, Section 16 in [1].

Lemma 3. *Let* A *be a* p*-group and* $B \leq A$. B *is an essential subgroup in* A *iff* $A\,[p] \subseteq B$.

A general definition of an $\mathfrak{X}$-envelope introduced by Enochs (1981) is given in [3] as:

Definition 1. *Let* $\mathfrak{X}$ *be any class of groups.* $I \in \mathfrak{X}$ *is called an* $\mathfrak{X}$*-envelope for an abelian group* A *if there is a homomorphism* $\phi : A \longrightarrow I$ *such that the following hold:*
1) *For any homomorphism* $f : A \longrightarrow X$ *with* $X \in \mathfrak{X}$, *there is a homomorphism* $g : I \longrightarrow X$ *such that* $f = g \circ \phi$.
2) *If an endomorphism* $h : I \longrightarrow I$ *is such that* $\phi = h \circ \phi$, *then* h *is an automorphism.*

Since the direct sum of injective (i.e. divisible) groups is injective and the sum of essential subgroups of summands is essential in the direct sum therefore by [4, Corollary 5.1.7], we have the following useful lemma which we will refer to later.

Lemma 4. *If* I_i *is an injective envelope for the groups* A_i *for every* $i \in J$, *then* $\bigoplus_{i\in J} I_i$ *is an injective envelope for* $\bigoplus_{i\in J} A_i$.

The following definitions and results can easily be derived parallel to the results given in [1, pp. 170–173] for pure-injective hull. Let G be a neat subgroup of A, and $K(G, A)$ denote the set of all subgroup $H \leq A$, such that $G \cap H = 0$ and $G + H/H$ is neat in A/H.

A group is called *neat-essential extension* of its subgroup G if G is neat in A, and if $K(G, A)$ consist of 0 only. A group A will be called a *maximal neat-essential extension* of G if A' with $A \subset A'$ is never a neat-essential extension of G. A *maximal neat-essential extension* of G is a minimal neat-injective group containing G as a neat subgroup. A is called a *neat-injective envelope* of G if A is a minimal neat-injective group containing a group G as a neat subgroup.

The equivalence of general definition of $\mathfrak{X}$-envelopes and the above definition of neat-injective envelopes is shown in the following proposition.

Proposition 1. *Let $\mathfrak{X}$ be the set of all neat-injective groups and A be any group. A group N containing A is a neat-injective envelope for A iff N is an $\mathfrak{X}$-envelope for A.*

The proof of the above Proposition is on similar lines as that of [3, Theorem 1.2.11].

We will often use the following Proposition from [2] .

Proposition 2. *Let A be a group. The neat-injective group N containing A as a neat subgroup is minimal if and only if the following two conditions hold:*

1) *$D(N)$, where $D(N)$ denotes the maximal divisible subgroup of a group N, is the divisible hull of the Frattini subgroup $F(A)$ of A;*

2) *N/A is divisible.*

3. MAIN RESULTS

Let A be any group and T be its torsion part. $T = \bigoplus_{p\in P} T_p$, where T_p is the p-component of T. For each T_p there is a basic subgroup B_p (see [1]) satisfying the following conditions:

1. B_p is a direct sum of groups isomorphic to $\mathbb{Z}(p^n)$ for some $n = 1, 2, \ldots$;
2. B_p is a pure subgroup of T_p;
3. T_p/B_p is divisible (therefore isomorphic to a direct sum of $\mathbb{Z}(p^\infty)$.

We shall assume that basic subgroups B_p, factor groups T_p/B_p, and torsion-free group A/T are given, and describe neat-injective envelopes of any arbitrary group A.

We begin with cyclic groups of prime power order. Simple groups $\mathbb{Z}(p)$ are themselves neat-injective.

Lemma 5. *A neat-injective envelope of $\mathbb{Z}(p^n)$ $(n \gneqq 1)$ is isomorphic to $\mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty)$.*

Proof. Let $i : \mathbb{Z}(p^n) \longrightarrow \mathbb{Z}(p^\infty)$ be the canonical inclusion and let a homomorphism $g : \mathbb{Z}(p^n) \longrightarrow \mathbb{Z}(p)$ satisfy $g(1) = 1$. Define

$$f : \mathbb{Z}(p^n) \longrightarrow \mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty), \qquad f(a) = (g(a), i(a)).$$

Since i is a monomorphism, f is also a monomorphism. If $p \mid f(a)$, i.e., $(g(a), i(a)) = p(x, y)$ for some $x \in \mathbb{Z}(p)$, $y \in \mathbb{Z}(p^\infty)$, then $g(a) = px = 0$. This means that $a \equiv 0 \pmod p$, that is $p \mid a$ in $\mathbb{Z}(p^n)$. Since $\mathbb{Z}(p^n)$ is divisible by every $q \neq p$, f is a neat monomorphism. We shall apply Proposition 2 to prove that $\mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty)$ is a neat-injective envelope of $f(\mathbb{Z}(p^n))$ or in other words $(\mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty), f)$ is a neat-injective envelope of $\mathbb{Z}(p^n)$. The Frattini subgroup $F(\mathbb{Z}(p^n)) = \langle p\rangle$ and $f(\langle p\rangle) \leq \mathbb{Z}(p^\infty)$ so $\mathbb{Z}(p^\infty)$ is an injective envelope of $f(\langle p\rangle)$ since the rank of $\mathbb{Z}(p^\infty)$ is 1. To show that $(\mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty))/f(\mathbb{Z}(p^n))$ is divisible, take any element $(x, y) + f(\mathbb{Z}(p^n))$. Since g is an epimorphism, $x = g(a)$ for some $a \in \mathbb{Z}(p^n)$. Then

$$(x, y) + f(\mathbb{Z}(p^n) = (0, y - i(a)) + f(a) + f(\mathbb{Z}(p^n)) = (0, y - i(a)) + f(\mathbb{Z}(p^n)$$

is divisible by every non-zero integer because $\mathbb{Z}(p^\infty)$ is a divisible group. So by Proposition 2, $\mathbb{Z}(p) \oplus \mathbb{Z}(p^\infty)$ is a neat-injective envelope of $f(\mathbb{Z}(p^n))$. □

Lemma 6. *Let p be a prime and $B = (\bigoplus_{i\in I} N_i) \bigoplus_{j\in J} K_j$, where each N_i is isomorphic to $\mathbb{Z}(p^n)$, for some $n \gneqq 1$, $K_j \cong \mathbb{Z}(p)$ for every $j \in J$, where I and J are any two disjoint index sets. Then the group $M = \mathbb{Z}_p^{(I\cup J)} \oplus \mathbb{Z}(p^\infty)^{(J)}$ is a neat-injective envelope for B.*

Proof. The group $\mathbb{Z}(p)$ is a neat-injective envelope of itself and $\mathbb{Z}(p)\oplus\mathbb{Z}(p^\infty)$ is a neat-injective envelope for $\mathbb{Z}(p^n)$. On the other hand, the direct sum of neat short exact sequences is neat by lemma 2, so we have a neat monomorphism $f : B \longrightarrow M$ such that $M/f(B)$ is a direct sum of divisible groups and therefore is divisible. It remains only to prove that M is neat-injective. But this follows from Lemma 1 since $p\mathbb{Z}(p)^{(I\cup J)} = 0$ and $\mathbb{Z}(p^\infty)^{(J)}$ is divisible. □

Now let T_p be any p-group. T_p contains a p-basic subgroup B_p (which is in fact a basic subgroup of T_p), since T_p is a p-group, (see [1, Theorem 32.3]) and B_p is a direct sum of groups isomorphic to $\mathbb{Z}(p^n)$ where $n = 1, 2, \ldots,$ is as required in the Lemma 8. So we know the structure of the neat-injective envelope of the basic subgroup B_p. On the other hand T_p/B_p is divisible, therefore it is a neat-injective envelope for itself.

The following proposition describes the neat-injective envelope of T_p in terms of those of B_p and T_p/B_p.

Proposition 3. *Let T_p be a p-group; M be a neat-injective envelope of the basic subgroup B_p of T_p and $K = T_p/B_p$. Then $M \oplus K$ is a neat-injective envelope of T_p.*

Proof. We have a neat-injective monomorphism $f : B_p \longrightarrow M$, the inclusion map $i : B_p \longrightarrow T_p$, and the natural epimorphism $g : T_p \longrightarrow K$ defined by $g(a) = a + B_p$. Since i is a pure (and therefore neat) monomorphism (see [1], Ch. 33) and M is neat-injective, there is a homomorphism $h : T_p \longrightarrow M$ such that $hoi = f$, i.e., $h\mid_{B_p} = f$. Define

$$e : T_p \longrightarrow M \oplus K, \qquad e(a) = (h(a), g(a)).$$

Clearly e is a homomorphism. If $e(a) = 0$, then $g(a) = 0$, i.e., $a \in B_p$. Since $f(a) = h(a) = 0$ and f is a monomorphism then $a = 0$. So e is a monomorphism. To prove that $e(T_p)$ is a neat subgroup of $M\oplus K$ it is sufficient to show that, for every element $e(a) \in e(T_p)$ divisible by p, a is divisible by p in T_p since all groups are p-groups. So let $e(a) = p(x, y)$. Then $h(a) = px$. Since K is divisible $g(a) = a + B_p = p(a_1 + B_p)$ for some $a_1 \in A$. Then $a - pa_1 = b \in B_p$ and $f(b) = h(b) = h(a) - hp(a_1) = p(x - h(a_1))$. Since $f(B_p)$ is neat in I, $b = pb_1$ for some $b_1 \in B_p$. Therefore $a = p(a_1 + b)$, that is a is divisible by p in A. To apply Proposition 6, we have to prove that:

1. The maximal divisible subgroup $D(M \oplus K)$ is a divisible envelope of the Frattini subgroup $F(e(T_p))$ of $e(T_p)$

2. $(M \oplus K)/e(T_p)$ is divisible.

For the first we use the following argument:

Since $M \cong \mathbb{Z}(p)^{(Q)}$ and $K \cong \mathbb{Z}(p^\infty)^{(S)}$, for some index sets Q and S(see Lemma 6) $p(M \oplus K)$ is divisible, therefore

$$F(e(T_p)) = pe(T_p) \subset p(M \oplus K) \subseteq D(M \oplus K).$$

To show that $pe(T_p)$ is essential in $D(M \oplus K)$, let $(m, k) \in D(M \oplus K)\,[p]$. Since K is divisible, $k = pk'$ for some $k' \in K$. Since g is an epimorphism, $k' = g(a)$ for some $a \in T_p$. Now $h(pa) = ph(a) \in pM \subseteq D(M)$, therefore $m - h(pa) \in D(M)$. Since M is a neat-injective envelope for B_p, $m - h(pa) \in f(pB_p)$, that is, $m - h(pa) = f(pb)$ for some $b \in B_p$. So

$$\begin{aligned} (m, k) &= (m - h(pa) + h(pa), pk') \\ &= (h(pb + pa), g(pa)) \\ &= e(pb + pa) \in pe(T_p) \end{aligned}$$

(note that $g(pb) = 0$). So by Lemma 3, $F(e(T_p)$ is essential in $D(M \oplus K)$.

For the second part let us consider the diagram

$$\begin{array}{ccccccccc}
 & & 0 & & 0 & & 0 & & \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
0 & \longrightarrow & B_p & \xrightarrow{i} & T_p & \xrightarrow{g} & K & \longrightarrow & 0 \\
 & & \downarrow f & & \downarrow e & & \downarrow 1_K & & \\
0 & \longrightarrow & M & \xrightarrow{j} & M \oplus K & \xrightarrow{p} & K & \longrightarrow & 0 \\
 & & \downarrow t & & \downarrow s & & \downarrow & & \\
0 & \longrightarrow & M/f(B_p) & \longrightarrow & (M \oplus K)/e(T_p) & \longrightarrow & 0 & \longrightarrow & 0 \\
 & & \downarrow & & \downarrow & & \downarrow & & \\
 & & 0 & & 0 & & 0 & &
\end{array}$$

where j is an inclusion map, $p : M \oplus K \longrightarrow K$ is a projection, and $t : M \longrightarrow M/f(B_p)$ and $s : M \oplus K \longrightarrow (M \oplus K)/e(T_p)$ are natural epimorphisms. Clearly all columns and first two rows are exact. By the 3×3 Lemma that last row is also exact, therefore $(M \oplus K)/e(T_p) \cong M/f(B_p)$ is divisible since M is a neat-injective envelope for B. Thus $(M \oplus K)$ is a neat-injective envelope for T_p by Proposition 2. □

Using the representation of any torsion group T as a direct sum of its p-components we can describe the neat injective envelope of T.

Proposition 4. *Let T be a torsion group and for each prime p, $M_p \oplus D_p$ be a neat-injective envelope for the p-component T_p of T with $pM_p = 0$ and divisible D_p. Then $M = (\prod_{p \in P} M_p) \oplus (\bigoplus_{p \in P} D_p)$ is a neat-injective envelope for T.*

Proof. Since D_p is an injective envelope for $F(T_p)$ for each prime p, $\bigoplus_{p\in P} D_p$ is an injective envelope for $F(T) = F(\bigoplus_{p\in P} T_p) = \bigoplus_{p\in P} F(T_p)$ by lemma 4. Since T_p is a neat subgroup of $M_p \oplus D_p$, we have $T = \bigoplus T_p$ is a neat subgroup of $K = (\bigoplus M_p) \oplus (\bigoplus D_p)$ by Lemma 2. By the properties of neat subgroups, $\bigoplus M_p$ is a neat subgroup of $\prod M_p$, so K is a neat-subgroup of M. Therefore T is a neat subgroup of M. $\prod M_p$ is neat-injective as a product of neat-injective groups, therefore M is neat-injective. Since $M/K \cong (M/T)/(K/T)$ and K/T is divisible, $M/T \cong K/T \oplus M/K$, so M/T is divisible if M/K is divisible. Clearly $M/K \cong \prod M_p / \bigoplus M_p$ and it is well-known (and easy to prove) that $\prod M_p / \bigoplus M_p$ is divisible, therefore M/K is also divisible. By Proposition 2, M is a neat-injective envelope for T. □

Now we shall try to describe neat-injective envelopes for torsion-free groups.

THEOREM 1. *Let S be a torsion-free group, D be an injective envelope of the Frattini subgroup $F(S)$ of S and for every prime p, α_p be the rank of the p-basic subgroup B_p of S. Then the neat-injective envelope of S is isomorphic to $E = D \oplus (\prod_p \mathbb{Z}(p)^{(I_p)}$, with $|I_p| = \alpha_p$.*

Proof. Since D is injective, the inclusion map $F(S) \longrightarrow D$ can be extended to a homomorphism $f : S \longrightarrow D$. By [1, A), p. 144], $S = B_p + pS$, therefore

$$S/pS = (B_p + pS)/pS \cong B_p/(B_p \cap pS) = B_p/pB_p.$$

Since S is torsion free, B_p is free and therefore $B_p/pB_p \cong \mathbb{Z}(p)^{(I_p)}$. This gives us an epimorphism $g_p : S \longrightarrow \mathbb{Z}(p)^{(I_p)}$ which is the composition of the natural epimorphism $S \longrightarrow S/pS$ and the isomorphism given above. Let us consider the homomorphism

$$h : S \longrightarrow E, \qquad h(a) = (f(a), g_2(a), g_3(a), g_5(a) \dots)$$

for every $a \in S$. If $h(a) = 0$ then $g_p(a) = 0$ for every prime p that is, $a \in pS$ for every p and so $a \in F(S)$. On the other hand $f(a) = 0$ and $f\ |_{F(S)}$ is a monomorphism, therefore $a = 0$. So h is a monomorphism. If $h(a)$ is divisible by some prime p then $g_p(a) = 0$, therefore $a \in pS$, that is a is divisible by p in S. So h is a neat monomorphism. Clearly $\prod \mathbb{Z}(p)^{(I_p)}$ is a reduced group so D is the maximal divisible subgroup of E, so the first condition of Proposition 2 is satisfied. Let $x = (d, b_2, b_3, b_5, \dots, b_p, \dots) + h(S)$ be any element of $E/h(S)$ and p be any prime. Then $d = pd'$ and $b_q = pb'_q$ for some $d' \in D$ and $b'_q \in \mathbb{Z}_q^{(I_p)}$ for all $q \neq p$. Therefore

$$\begin{aligned} x &= (d, b_2, b_3, b_5, \dots, 0, \dots) + (0, \dots, 0, b_p, 0, \dots) + h(S) \\ &= p(d', b'_2, \dots, 0, \dots) + h(S). \end{aligned}$$

So $E/h(S)$ is divisible. By Proposition 2, E together with the monomorphism h is a neat-injective envelope for S. □

Knowing the neat-injective envelopes of torsion and torsion-free groups, we are now able to describe neat-injective envelopes of arbitrary group.

THEOREM 2. *Let A be any group, T be its torsion part, $S = A/T$, I be a neat-injective envelope for T, $M = \prod_{p\in P} S/pS$, and D be an injective envelope for $F(A)/F(T)$. Then a neat-injective envelope for A is isomorphic to $J = I \oplus D \oplus M$.*

Proof. Since T is a pure subgroup and therefore a neat subgroup and I is a neat-injective group, we have a homomorphism $f : A \longrightarrow I$ whose restriction on T is an inclusion map. Since D is divisible, we have a homomorphism $g : A \longrightarrow D$ whose restriction on $F(A)$ is the composition of natural homomorphism $\sigma : F(A) \longrightarrow F(A)/F(T)$ and the inclusion map $F(A)/F(T) \longrightarrow D$. Also we have a homomorphism $h : A \longrightarrow M$ defined by $h(a) = (\dots, (a+T) + pS, \dots)$. Now define

$$u : A \longrightarrow J, \qquad u(a) = (f(a), g(a), h(a)).$$

Clearly u is a homomorphism and J is neat-injective.

If $u(a) = 0$, then $h(a) = 0$, therefore $a + T \in pS$ for every $p \in P$. So for each $p \in P$, $a + T = p(a' + T)$ for some $a' \in A$. Then $t = a - pa' \in T$ and since $0 = f(a) = f(t) - pf(a')$ from the neatness of T in I we conclude $t = pt'$ for some $t' \in T$. Then $a = p(a' + t') \in pA$. Since this holds for every $p \in P$, $a \in F(A)$. Now $g(a) = 0$, therefore $a \in F(T) \subset T$. Since $f\mid_T$ is an inclusion map, the equality $f(a) = 0$ implies that $a = 0$. So u is a monomorphism.

Let $u(a)$ be divisible by some $p \in P$. Then $(a+T)+pS = pS$, i.e., $a+T \in pS$ and therefore $t = a - pa' \in T$ for some $a' \in A$ as above. Since $f(a)$ is divisible by p, $f(t) = f(a) - pf(a')$ is divisible by p. Therefore t is divisible by p since T is neat in I. Then $a = t + pa'$ is divisible by p in A. So $u(A)$ is neat in J. $u(F(A)) \leq F(J) = D(J)$.

To prove that $u(F(A)) \trianglelefteq D(J)$, let $(x, y, 0) \in D(J)$. If $y = 0$ and $x \neq 0$ then $\langle x\rangle \cap F(T) \neq 0$, therefore $\langle (x, 0, 0)\rangle \cap u(F(A)) \neq 0$. Let $y \neq 0$. Since $g(F(A)) \trianglelefteq D$ there exists $0 \neq ny \in g(F(A))$, i.e., $ny = g(a)$ for some $a \in F(A)$. Clearly $h(a) = 0, f(a) \in F(I) = D(I)$. Since $x \in D(I)$, we have $nx - f(a) \in D(I)$. If $nx - f(a) = 0$, then $0 \neq n(x, y, 0) = u(a) \in u(A)$. If $nx - f(a) \neq 0$ then $0 \neq m(nx - f(a)) = t \in T$ since $T \trianglelefteq I$. Now

$$u(ma + t) = (mf(a) + mnx - mf(a), mny, 0) = mn(x, y) \in u(A).$$

If $mn(x, y) = 0$, then $g(ma) = mny = 0$, therefore $ma \in F(T)$. But then $a \in F(T)$ and $ny = g(a) = 0$, contradiction, so $mn(x, y) \neq 0$. Thus $u(F(A)) \trianglelefteq D$.

To prove that $J/u(A)$ is divisible, let $(x, y, z) \in J$ and $p \in P$. We can construct an element $z' \in M$, divisible by p such that $z - z' = h(a)$ for some $a \in A$. Since I/T is divisible, there is an element $t \in T$ such that $x - f(a) - t$ is divisible by p. Then $(x, y, z) - u(a + t) = (x - f(a) - t, y - g(a + t), z')$ is divisible by p, therefore $(x, y, z) + u(A)$ is divisible by p in $J/u(A)$, i.e., $J/u(A)$ is divisible. So by Proposition 2, J is a neat-injective envelope for $u(A)$. □

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