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OPTIMAL CONTROL OF A MATHEMATICAL MODEL FOR THE 2014 EBOLA OUTBREAK IN WEST AFRICA

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Abstract. Ebola hemorrhagic fever is a highly infectious and lethal disease that poses serious public health risks in Africa and even countries beyond the African continent. The main goal of this study is to develop a theoretical optimal control treatment of Ebola. The aim of the mathematical model used herein is to make the number of the infectious individuals decrease and the number of recovered individuals increase, while administering an efficient medical treatment (vaccination / medication). Pontryagin's classical control theory is applied to a SEIR mathematical model of Ebola infection characterized by a system of nonlinear differential equations with the following unknown functions: the susceptible individuals, exposed individuals, infectious individuals and recovered individuals. An optimal control strategy is derived for 2014 Ebola outbreaks in Guinea, Sierra Leone, Liberia and Nigeria.

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Key words. Ebola, system of nonlinear differential equations, optimal control, state equations, adjoint equations.

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