

ON SOME NONZERO RINGEL-HALL NUMBERS  
IN TAME CASES

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**Abstract.** Let  $k$  be a finite field and consider the finite dimensional path algebra  $kQ$  where  $Q$  is a quiver of tame type i.e. of type  $\tilde{A}_n, \tilde{D}_n, \tilde{E}_6, \tilde{E}_7, \tilde{E}_8$ . Let  $\mathcal{H}(kQ)$  be the corresponding Ringel-Hall algebra. We are going to study the Ringel-Hall numbers of the form  $F_{XP}^{P'}$  with  $P, P'$  preprojective indecomposables of defect -1 and  $F_{IX}^{I'}$  with  $I, I'$  preinjective indecomposables of defect 1. More precisely we will give necessary conditions for the module  $X$  such that these Ringel-Hall numbers are nonzero.

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**Key words.** Tame hereditary algebra, Ringel-Hall algebra, Ringel-Hall numbers.

REFERENCES

- [1] ASSEM, I., SIMSON, D. and SKOWRONSKI, A., *Elements of Representation Theory of Associative Algebras, Volume 1: Techniques of Representation Theory*, LMS Student Texts, **65**, Cambridge Univ. Press, 2006.
- [2] AUSLANDER, M., REITEN, I. and SMALØ, S., *Representation Theory of Artin Algebras*, Cambridge Stud. in Adv. Math., **36**, Cambridge Univ. Press, 1995.
- [3] DLAB, V. and RINGEL, C. M., *Indecomposable representations of graphs and algebras*, Mem. Amer. Math. Soc., **173**, 1976.
- [4] RINGEL, C. M., *Tame algebras and Integral Quadratic Forms*, Lect. Notes Math., **1099**, Springer, 1984.
- [5] ZHANG, P., *Composition algebras of affine type*, J. Algebra, **206** (1998), 505–540.

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