

ON COMMUTATIVITY OF 2-TORSION FREE \*-PRIME RINGS  
WITH GENERALIZED DERIVATIONS

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**Abstract.** Let  $(R, *)$  be a 2-torsion free \*-prime ring with involution  $*$ ,  $I \neq 0$  a \*-ideal of  $R$ . An additive mapping  $F: R \rightarrow R$  is called a generalized derivation on  $R$  if there exists a derivation  $d: R \rightarrow R$  such that  $F(xy) = F(x)y + xd(y)$  holds for all  $x, y \in R$ . In the present paper, we prove the commutativity of a \*-prime ring  $R$  admitting generalized derivations satisfying several conditions, but associated with a derivation commuting with  $*$ .

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